BITL3 | **Math Lab**

Worksheet 14

4 April 2022

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1. **Warm up:** Answer the following True / False questions.

   (a) Every geometric series converges.

   (b) Every alternating series converges.

   (c) If $\sum_{n=0}^{\infty}(-1)^n a_n$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.

   (d) If $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty}(-1)^n a_n$ converges.

2. Consider the series $\sum_{n=1}^{\infty} \frac{a^n n!}{n^n}$ for some $a \in \mathbf{R}_{>0}$.

   (a) Show that the series converges for $0 < a < e$ and diverges for $a > e$.

   (b) For $n \geqslant 2$, use the inequality

   $$\ln(1) + \ln(2) + \cdots + \ln(n-1) < \int_1^n \ln(x)\ dx < \ln(2) + \cdots + \ln(n)$$

   to show that $\frac{n^n}{e^{n-1}} < n! < \frac{(n+1)^{n+1}}{e^n}$.

   (c) Use part (c) to determine if the series converges if $a = e$.

3. Determine if each of the following series is alternating. If it is, determine if it is absolutely or conditionally convergent.

   (a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(1-n)}{n-2n^2}$ (b) $\sum_{m=2}^{\infty} \frac{(-1)^m}{\sqrt{m} + \frac{m^2}{2}}$ (c) $\sum_{k=1}^{\infty} \frac{\sin(k\pi/4)}{k}$

4. Use any tests you know to determine if the following series converge or diverge.

   (a) $\sum_{n=1}^{\infty} \frac{n+1}{n}$ (c) $\sum_{n=1}^{\infty}(-1)^n \frac{n^3}{3^n}$ (e) $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$

   (b) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+4}$ (d) $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{3n}}$ (f) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$

5. Use any tests you know to determine if the following series converge or diverge.

   (a) $\sum_{n=1}^{\infty} \frac{n+1}{n^3}$ (c) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ (e) $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

   (b) $\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$ (d) $\sum_{n=1}^{\infty} \sin(1/n^2)$ (f) $\sum_{n=1}^{\infty} \frac{n!}{4^n n^3}$