BITL3 | # Worksheet 13 | **Math Lab**

28 March 2022

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1. **Warm up:** Answer the following True / False questions.

   (a) $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is a geometric series

   (b) The series $\sum_{n=1}^{\infty} \frac{n}{n^p}$ converges for all $p \in \mathbf{R}$

   (c) If the limit of $\{a_n\}_{n=1}^{\infty}$ is 0, then $\sum_{n=0}^{\infty} a_n$ converges.

   (d) If $\sum_{n=0}^{\infty} a_n$ converges, then the limit of $\{a_n\}_{n=1}^{\infty}$ is 0.

2. Determine if each of the following geometric series converge or diverge. If they converge, find the value of the series.

   (a) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$ (b) $\sum_{n=0}^{\infty} 2^{2n} 4^{3n+1} e^{8-8n}$ (c) $\sum_{n=0}^{\infty} (-1)^n \pi^{3-n} 2^{n+1} - \left(\frac{2}{3}\right)^{2n}$ (d) $\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n 2^{2-3n}$

3. Recall the integral test says that for a non-increasing function f, the sum $\sum_{n=N}^{\infty} f(n)$ converges if and only if the integral $\int_N^{\infty} f(x)\, dx$ is finite.

   (a) Show that $\sum_{n=2}^{\infty} \left(\frac{e}{n}\right)^n$ converges.

   (b) Show that $\int_1^{\infty} \frac{e^y}{y^y}\, dy$ converges.

   (c) Determine whether or not $\sum_{n=3}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$ converges.

4. Let $f(n) = (9n^2 - 3n - 2)^{-1}$, and let $L = \sum_{n=1}^{\infty} f(n)$.

   (a) Estimate the error $L - S_k$, for S_k the partial sum $S_k = \sum_{n=1}^{k} f(n)$, for any $k \in \mathbf{N}$.

   (b) Compute the limit $\lim_{k \to \infty} S_k$.

5. Use the integral test for series to determine if each of the folowing series converge or diverge. If they converge, find how many terms are necessary to approximate the value of the series to 3 decimal places.

   (a) $\sum_{n=2}^{\infty} \frac{1}{n \ln^4(n)}$ (b) $\sum_{n=1}^{\infty} \frac{1}{4n^2 + 9}$ (c) $\sum_{n=1}^{\infty} \frac{3}{n^2 + 7n + 10}$