BITL3 | # Worksheet 12 | **Math Lab**

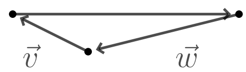
21 March 2022

1. **Warm up:** Answer the following True / False questions.

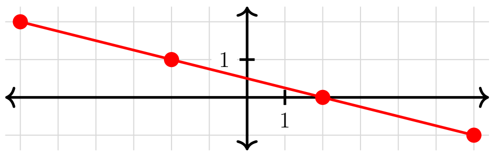
   (a) If the scalar product of two nonzero vectors is 0, then they are perpendicular.

   (b) It is not possible for every two from a set of three vectors $\{\vec{v}, \vec{w}, \vec{z}\}$ to be perpendicular to each other.

   (c) In the diagram below, the unmarked vector is $\vec{v} + \vec{w}$.



   (d) In the diagram below, the red line is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ and the marked points are for $t = -3, -2, -1, 0$.



2. Let ℓ be the line going through the points $a = (1, 5, 2)$ and $b = (6, 6, -3)$.

   (a) Find three different ways to write the equation for ℓ, by using different values of t.

   (b) For each of the three expressions for ℓ in part (a), find the equations of the plane going through $\ell(0)$ and to which ℓ is normal.

   (c) There are infinitely many planes going through ℓ, but does every vector (x, y, z) lie in one of these planes? Why or why not?

3. Consider the planes $6x + 2y - z = 3$ and $-2x + 3y - z = 9$.

   (a) What are the normal vectors to each of these planes? Find the angle between them.

   (b) Find the equation of the line of intersection of the two planes.

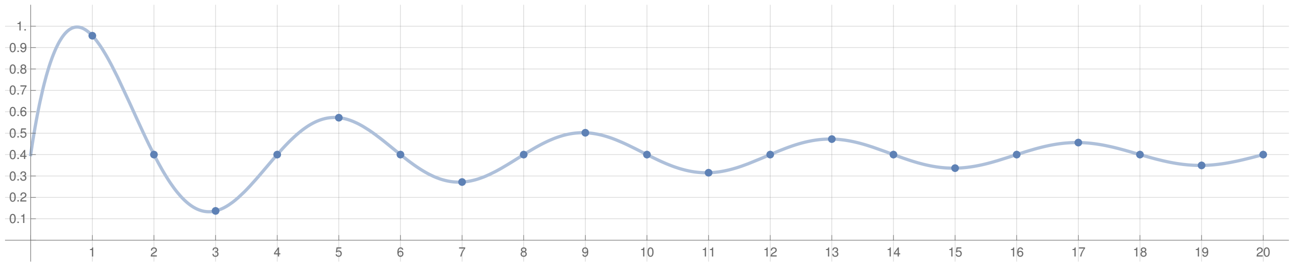
   (c) The two planes intersect at $p = (-1/2, 2, -2)$.

      i. For the first plane, find a vector $\vec{v}$ so that $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{p} + t\vec{v}$ is in the plane for all t.

      ii. For the second plane, find a vector $\vec{w}$ so that $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \vec{p} + t\vec{w}$ is in the plane for all t.

      iii. Find the angle between $\vec{v}$ and $\vec{w}$. How does your answer compare to part (a)? Should your answer be similar?

4. Consider the first 20 terms of a sequence $\{a_n\}_{n=1}^{\infty}$ below, along with a function $f(x)$ with $f(n) = a_n$ for every $n \in \mathbf{N}$.



(a) Does this sequence look convergent? If so, what do you think the limit will be?

(b) Is this sequence monotonic? Is it bounded?

(c) For $\epsilon_1 = \frac{3}{10}$ and $\epsilon_2 = \frac{1}{10}$, find $N_1, N_2 \in \mathbf{N}$ so that the epsilon definition of the limit satisfied, respectively.

(d) Consider the sequence $\{b_n\}_{n=1}^{\infty}$ where $b_n = a_{2n}$ for every $n \in \mathbf{N}$. Is this sequence monotonic? Bounded? Convergent?

5. Knowing the sequence from Question 4 is $a_n = \frac{\sin(\pi n/2)}{n+\frac{4}{5}} + \frac{2}{5}$, answer the following questions.

(a) Compute $L = \lim_{n\to\infty} a_n$.

(b) For any $\epsilon > 0$, find $N \in \mathbf{N}$ so that the epsilon definition of the limit will be satisfied.

(c) For $\epsilon_1 = \frac{3}{10}$ and $\epsilon_2 = \frac{1}{10}$, find $N_1, N_2 \in \mathbf{N}$ using part (b) above. How does this compare with your estimate in part (c) of Question 4?