Worksheet 11

14 March 2022

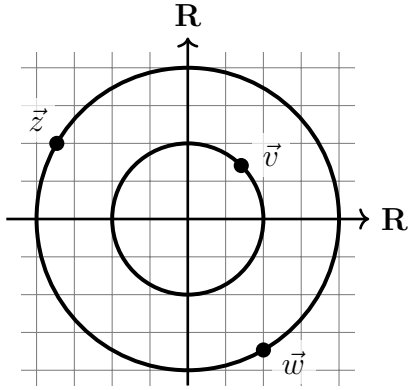
1. **Warm up:** Answer the following True / False questions, for $\vec{v}, \vec{w} \in \mathbf{R}^3$ two vectors.
   (a) If $|\vec{v}| = |\vec{w}|$, then $\vec{v} = \vec{w}$.
   (b) If $|\vec{v}| = k$, then $|2\vec{v}| = 2k$.
   (c) If $\vec{v} \bullet \vec{w} = k$, then $(2\vec{v}) \bullet \vec{w} = 2k$.
   (d) If the angle between $\vec{v}$ and $\vec{w}$ is 0, then $\vec{v} \bullet \vec{w} = 0$.
   (e) The scalar product of $\vec{v}$ and $\vec{w}$ is another vector in $\mathbf{R}^3$.

2. Consider the points $a = (0, 0, 4)$, $b = (-1, 4, 2)$, $c = (0, -3, 2)$, and $d = (1, -2, -3)$ in $\mathbf{R}^3$.
   (a) Compute the vectors $\overrightarrow{ab}$ and $\overrightarrow{cd}$.
   (b) Find a point $e \in \mathbf{R}^3$ so that $|\overrightarrow{ae}| = |\overrightarrow{be}|$.
   (c) Find a point $f \in \mathbf{R}^3$ so that $|\overrightarrow{af}| = |\overrightarrow{bf}| = |\overrightarrow{cf}|$.
   (d) Does there exist a point $g \in \mathbf{R}^3$ so that $|\overrightarrow{ag}| = |\overrightarrow{bg}| = |\overrightarrow{cg}| = |\overrightarrow{dg}|$? If yes what is it? If no, why not?

3. Let $\vec{a} = (1, 1, 1)$ and $\vec{b} = (1, 1, 0)$ be vectors in $\mathbf{R}^3$.
   (a) Compute the magnitudes of $\vec{a}$ and $\vec{b}$.
   (b) What is the angle θ between $\vec{a}$ and $\vec{b}$?
   (c) Find the unique vector of magnitude 1 that forms an angle of $\frac{\theta}{2}$ with both $\vec{a}$, $\vec{b}$.
   (d) Find the two unique vectors of magnitude 1 that are perpendicular to both $\vec{a}$, $\vec{b}$.

4. Consider the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and the vectors $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.
   (a) What are the magintudes $|\vec{v}|$ and $|\vec{w}|$? What is the angle between $\vec{v}$ and $\vec{w}$?
   (b) Find the magnitude of $A\vec{v}$, $A^2\vec{v}$, and $A^3\vec{v}$. What will the magnitude of $A^{1000}\vec{v}$ be?
   (c) Repeat part (b) above for $\vec{w}$ instead of $\vec{v}$. Does the same thing happen? Why or why not?

5. Consider the vectors $\vec{v}, \vec{w}, \vec{z}$ as in the picture below. The inner circle has radius 1, and the outer circle has radius 2.



$\vec{v}$ makes an angle of $\frac{\pi}{2}$ with the positive x-axis

$\vec{w}$ makes an angle of $\frac{\pi}{3}$ with the positive x-axis

$\vec{z}$ makes an angle of $\frac{\pi}{6}$ with the negative x-axis

   (a) Compute the coordinates of the vectors $\vec{v}, \vec{w}, \vec{z}$. That is, express each as a pair of numbers in the horizontal and vecrtical directions.
   (b) The three vectors form a triangle. Find the lengths of the sides of this triangle.
   (c) **Bonus:** If the three vectors were to change angle (but not length), what do you think would be the largest possible triangle they could form?