BITL3 **Math Lab**

Worksheet 9

28 February 2022

Recall the following definitions:

- A **matrix** is a collection of $m \cdot n$ numbers, presented as an array. A matrix with m **rows** and n **columns** is called an "$m \times n$ matrix", or an "m by n matrix".
- A **vector** is an $m \times 1$ matrix for some $m \in \mathbf{N}$. That is, a vector is a matrix with a single column. Vectors are often denoted with arrows, like $\vec{x}$ and $\vec{y}$.

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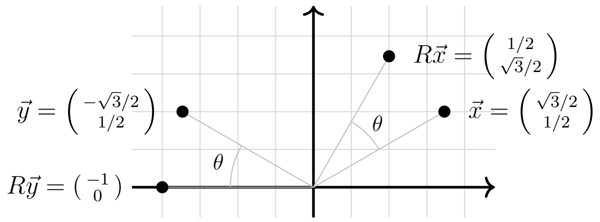
1. **Warm up:** Find vectors $\vec{x}$ that make the following equalities $A\vec{x} = \vec{b}$ true.

   (a) $\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 5 \\ -6 \end{bmatrix}}_{\vec{b}}$

   (b) $\underbrace{\begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 4 \\ -1 \end{bmatrix}}_{\vec{b}}$

2. The *rotation matrix* $R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ rotates every 2×1 vector counterclockwise by an angle of θ. For example, if $\theta = \frac{\pi}{6}$, then:



   (a) Calculate R^2 by matrix multiplication.

   (b) What will R^n be, for any $n \in \mathbf{N}$?

   (c) Find a matrix S such that $S^2 = R$ (the square root of R). *Hint: think geometrically.*

3. This question is about the $n \times n$ *identity matrix*, denoted I_n.

   (a) Let $A = \left[\begin{smallmatrix} 5 & 0 \\ 0 & 7 \end{smallmatrix}\right]$. Find a matrix B so that $AB = I_2$. Will $BA = I_2$ as well?

   (b) Let $A = \left[\begin{smallmatrix} 5 & 14 \\ 0 & 7 \end{smallmatrix}\right]$. Find a matrix B so that $AB = I_2$.

   (c) Let $A = \left[\begin{smallmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{smallmatrix}\right]$. Find a matrix B so that $AB = I_3$.

   (d) For the rotation matrix R from question 2 above, find a matrix R' so that $R'R = I_2$.

4. Suppose that A, B are 3×3 matrices and that $A\vec{x} = B\vec{x}$ for every 3×1 vector $\vec{x}$. Show that $A = B$.

5. For each of the following systems of linear equations, find at least one solution in the appropriate spaces. If no solutions exist, say so.

   (a) $5 + 4x_1 = 0$ in $\mathbf{R}^2$

   (b) $\begin{aligned} 1 + 2x_1 &= 0 \\ 1 + 3x_1 &= 0 \end{aligned}$ in $\mathbf{R}^1$

   (c) $\begin{aligned} 1 + 2x_1 &= 0 \\ 1 + 3x_1 &= 0 \end{aligned}$ in $\mathbf{R}^2$

   (d) $\begin{aligned} 2 + 2x_1 &= 0 \\ 1 - x_2 &= 0 \\ -2 + \pi x_3 &= 0 \end{aligned}$ in $\mathbf{R}^4$