Worksheet 7

14 February 2022

1. **Warm up:** Answer the following True / False questions.

   (a) The integral of an odd function is always zero.

   (b) A rational function can have e or ln appearing in it.

   (c) The epxression $\dfrac{4x^2 + 3x - \frac{2}{3x} + 1}{3x - \frac{7x+2}{x^2}}$ is a rational function.

2. Let $a \neq b$ be fixed real numbers. Prove the general formula
$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b} \ln\left(\frac{x-a}{x-b}\right) + C.$$

3. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{9 - x^2}{x - 3}\, dx$

   (b) $\displaystyle\int \frac{dx}{x(x^2 + x)}$

   (c) $\displaystyle\int \frac{3x^2 - 2}{x - 4}\, dx$

   (d) $\displaystyle\int \frac{7x}{x^3 - 4x^2 + 4x}\, dx$

4. Evaluate the following slightly more complicated integrals.

   (a) $\displaystyle\int \frac{dx}{x^2 - 7x + 10}$

   (b) $\displaystyle\int \frac{3x + 6}{x^2(x - 1)(x - 3)}\, dx$

   (c) $\displaystyle\int \frac{5x - 1}{x^2 - 2x - 5}\, dx$

   (d) $\displaystyle\int \frac{x + 1}{x^2 + x + 5}\, dx$

5. Evaluate the following integrals, which are:

   (a) Type 1 and Type 2 integrals, for $A, B, a, b \in \mathbf{R}$ and $n \in \mathbf{N}_{>1}$

       i. $\displaystyle\int \frac{A}{ax + b}\, dx$
       ii. $\displaystyle\int \frac{A + B}{(ax + b)^2}\, dx$
       iii. $\displaystyle\int \frac{(A + B)^2}{(ax + b)^n}\, dx$

   (b) Type 3 and mixed type

       i. $\displaystyle\int \frac{1}{x^2 - x - 2}\, dx$
       ii. $\displaystyle\int \frac{1}{x^2 + x - 2}\, dx$
       iii. $\displaystyle\int \frac{1}{x^2 + x + 2}\, dx$

   (c) Type 4 and mixed type

       i. $\displaystyle\int \frac{2x + 1}{(x^2 - x - 2)^2}\, dx$
       ii. $\displaystyle\int \frac{2x + 1}{(x^2 + x + 2)^2}\, dx$