1. **Warm up:** Prove an alternative version of the integration by parts formula:

$$\int f(x)g(x)\ dx = f(x)\int g(x)\ dx - \int f'(x)\int g(x)\ dx\ dx.$$

2. Find $f(x)$ if you know that $\int f(x)e^x\ dx = f(x)e^x - \int \frac{e^x}{x}\ dx.$

3. Find $\int (\ln(x))^k\ dx$ for any positive integer k.

   Hint: Find a pattern by computing the integral for small k.

4. (a) What is a *polynomial*?

   (b) What is the *degree* of a polynomial?

   (c) Show by differentiation that if $P_n(x)$ is a polynomial of degree n which satisfies the equation $P_n(x) + P_n'(x) = x^n$, then $\int x^n e^x dx = P_n(x)e^x + C.$

5. Evaluate the following integrals. Be careful in choosing which method to use.

   (a) $\int x \ln(x)\ dx$

   (b) $\int x \cos(4x)\ dx$

   (c) $\int e^{4x} \cos(3x)\ dx$

   (d) $\int x^2 \sin(x)\ dx$

   (e) $\int x \sin(3x + 4)\ dx$

   (f) $\int \sin(3x)\cos(5x)\ dx$