BITL3

Worksheet 5

31 January 2022

Math Lab

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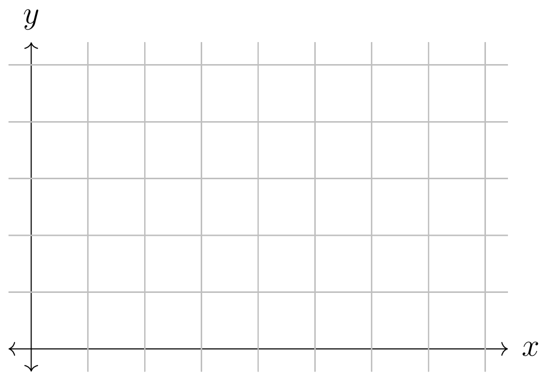
1. **Warm up:** Answer the following True / False questions.
   (a) If f crosses the x-axis on $[a, b]$, then it is not possible to compute the solid of revolution of f on $[a, b]$.
   (b) Any 3-dimensional shape can be described as a solid of revolution of some function on some interval.
   (c) If $f(x) < 0$ on $[a, b]$, then the volume of the solid of revolution of f on $[a, b]$ will be negative.
   (d) The mean value theorem for integrals applies to all funtions defined on an interval $[a, b]$.
2. Let $f(x) = x^2 \ln(x)$ and $g(x) = 4 \ln(x)$.
   (a) Where do these two curves intersect?
   (b) Express the area bounded by these two curves as a definite integral.
   (c) Evaluate this integral.
3. Let $f(x) = \sqrt{x}$, $g(x) = x^2$ and $h(x) = 2x$ for $0 \leqslant x \leqslant 1$.
   (a) Draw the graphs of the functions on the given interval on the grid.
   (b) Find the area of the region with all three of these functions as its boundary on this interval.
   (c) Find the volume of revolution of this area around the x-axis.



4. A sharpened pencil is 0.5cm wide, 20cm long, and its tip is sharpened to an angle of $\pi/3$.
   (a) Using solids of revolution, write down the integral that gives the total volume of the sharpened pencil.
   (b) Evaluate the integral to find the volume of this pencil.
   (c) Using geometry, describe the pencil as two simpler shapes, and calculate their volume without using solids of revolution.
5. Someone drives a car from city A to city B in 1 hour and 30 minutes. The speed limit is 100km/h, and the distance between the cities is 160 km.
   (a) What is the average speed of the car during the trip?
   (b) Using the mean value theorem for integrals, prove that there must have been some point in time at which the car was driving faster than the speed limit.
   (c) Suppose the speed limit is 120km/h. Is it possible that the car was driving faster than this speed limit? Why or why not? Explain using integrals.