BITL3 | **Math Lab**

Worksheet 4

24 January 2022

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1. **Warm up:** Find errors in the following two solutions, and complete them correctly.

   $\int \cos(x)\sin(x)\, dx$
   Let $u = \cos(x)$.
   Then $du = -\sin(x)dx$.
   So the integral is $-\int \cos(x)\, du$.
   This simplifies to $-\sin(x) + C$.

   $\int_{-2}^{2} \frac{|x|}{x}\, dx$
   Note that $f(x) = \frac{|x|}{x} = \begin{cases} -1 & x<0, \\ 1 & x>0. \end{cases}$
   Hence $F(x) = \begin{cases} -x & x<0, \\ x & x>0. \end{cases}$
   So the integral is $F(2) - F(-2)$.
   This becomes to $2 - 2 = 0$.

2. Evalute the following integrals using integration by substitution.

   (a) $\int \frac{x}{\sqrt{x^2+9}}\, dx$

   (b) $\int x^2 \sin(x^3)\, dx$

   (c) $\int \sin^5(x)\cos(x)\, dx$

   (d) $\int (x^7+2)(x^8+16x-5)^4\, dx$

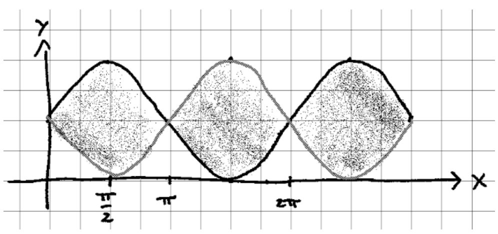
   (e) $\int \frac{2x-1}{x^2-x}\, dx$

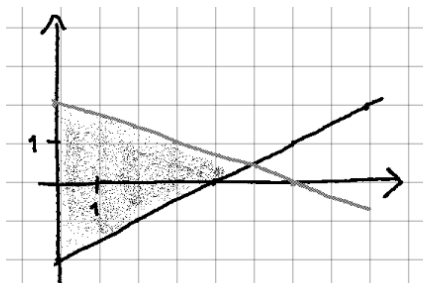
   (f) $\int \frac{x^2 e^{\sqrt{x^3-3}}}{\sqrt{x^3-3}}\, dx$

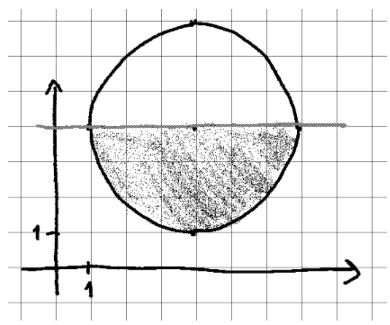
   (g) $\int \frac{x}{1+x}\, dx$

   (h) $\int \frac{x^8}{x^3+4}\, dx$

3. For each of the following functions and shaded regions, find equations of the functions and give a formula using definite integrals that computes the area. Do not evaluate the formula.







4. Find the area between the curves:

   (a) $y = x^2$, $y = a|x|$, and $y = a^2$, where $a > 0$

   (b) $y = x^n$, $y = x^m$, and $y = m + n$, where $m, n \in \mathbf{N}$ such that $1 \leqslant m < n$

5. **Bonus:** Suppose that f has an inverse function f^{-1} (so $f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$ for all x, y). Show that

$$\int_a^b f(x)\ dx + \int_{f(a)}^{f(b)} f^{-1}(x)\ dx = bf(b) - af(a).$$

Hint: First show that $\displaystyle\int_{f(a)}^{f(b)} f^{-1}(x)\ dx = \int_a^b yf'(y)\ dy$ by using the substitution $x = f(y)$.