BITL3 **Math Lab**

Worksheet 3

17 January 2022

Recall the fundamental theorems of calculus (FTC). Both assume that f is continuous on (a, b).

1st FTC: $\dfrac{d}{dx}\displaystyle\int_a^x f(t)\,dt = f(x)$ for any $x \in (a, b)$.

2nd FTC: $\displaystyle\int_a^b f(t)\,dt = F(b) - F(a)$ for any antiderivative F of f.

The 2nd fundamental theorem of calculus is often called the **Newton–Leibniz formula**.

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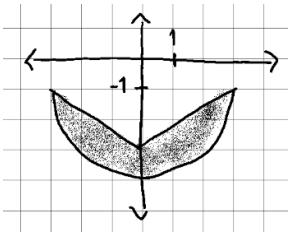
1. **Warm up:** Answer the following True / False questions.
   (a) A function has a unique antiderivative.
   (b) Even functions always have odd functions as antiderivatives.
   (c) If $f(a) > 0$ for some number a, then $F(a) > 0$ as well, for F an antiderivative of f.

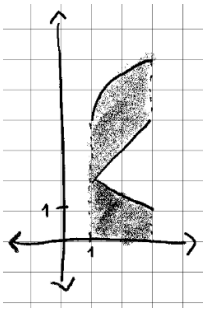
2. Compute the following integrals using the Newton–Leibniz formula.
   (a) $\displaystyle\int_1^4 \left(3\sqrt{x} - \frac{2}{x}\right)\,dx$
   (b) $\displaystyle\int_0^\pi -2\cos(x)\,dx$
   (c) $\displaystyle\int_1^0 e^x\,dx$
   (d) $\displaystyle\int_{-3}^2 (3x^2 + 4x)\,dx$

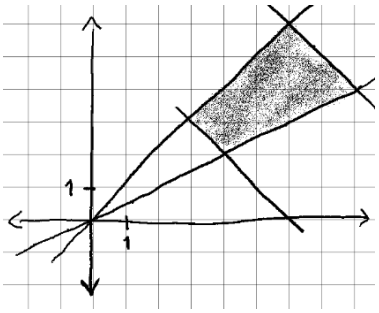
3. Suppose that $\int_1^4 f(x)\,dx = 8$ and $\int_1^6 f(x)\,dx = 5$. Evaluate the following definite integrals.
   (a) $\displaystyle\int_4^1 -3f(x)\,dx$ (b) $\displaystyle\int_4^4 5f(x)\,dx$ (c) $\displaystyle\int_4^6 f(x)\,dx$ (d) $\displaystyle\int_6^4 2f(x)\,dx$

4. (a) Describe, in your own words, what is an even function and what is an odd function.
   (b) Do functions that are neither even nor odd exist? If no, why? If yes, give an example.
   (c) Are the two expressions the same or not? Why? $\displaystyle\int_{-1}^1 \frac{1}{x^2}\,dx$ and $\displaystyle 2\int_0^1 \frac{1}{x^2}\,dx$

5. Express the following shaded areas as integrals.

(a) 

(b) 

(c) 

6. Draw pictures and use areas of triangles / rectangles / circles to evaluate the following integrals.
   (a) $\displaystyle\int_{-1}^2 \sqrt{4 - (x-1)^2} + 2\,dx$
   (b) $\displaystyle\int_2^6 |x - 3| + 2 - \frac{1}{2}(x+1)\,dx$