BITL3 **Math Lab**

Worksheet 2

10 January 2022

1. **Warm up:** Answer the following True / False questions.
   (a) If $f(x)$ is constant, then the derivative of f is constant.
   (b) If $g(x)$ is constant, then the indefinite integral of g is constant.
   (c) The expressions $2\int f(x)\,dx$ and $\int 2f(x)\,dx$ are the same.
   (d) The expressions $|\int f(x)\,dx|$ and $\int |f(x)|\,dx$ are the same.
   (e) The expressions $(\int f(x)\,dx)^2$ and $\int (f(x))^2\,dx$ are the same.

2. For each of the following sets of conditions, find $f(x)$.
   (a) $f(2) = \frac{5}{2}$, $f'(x) = 4 + 6x + 24x^2$
   (b) $f(0) = 3$, $f'(\frac{\pi}{3}) = 2$, $f''(x) = \sin(x) + \cos(x)$
   (c) $f(1) = 0$, $f(0) = 2$, $f'(x) = e^{2x} - x^{-2e} + Cx$ for some $C \in \mathbf{R}$

3. Let $f(x) = x + x^2$, and let $g(x) = f(3x - 2)$.
   (a) Find an antiderivative of $f(x)$.
   (b) Find an antiderivative of $g(x)$ using the "$kx + b$" integration rule.
   (c) Find an antiderivative of $g(x)$ by expanding $g(x)$ and using the power rule.

4. Compute antiderivatives of the following functions using the $kx + b$ integration rule.
   (a) $4x + 2^{4x+2}$ (b) $\sin^2(1-x) + \cot^2(1-x) + \cos^2(1-x)$ (c) $\dfrac{3x - 22}{x - 9}$

5. Evaluate the following strange-looking integrals. Simplify and expand first!
   (a) $\displaystyle\sum_{k=1}^{20} \left(\int x^k - x^{k+1}\,dx \right)$ (b) $\displaystyle\int \left(\sum_{\ell=1}^{30} x^{3\ell-8} \right) dx$ (c) $\displaystyle\int \frac{1}{1 + \frac{1}{1+\frac{1}{1+x}}}\,dx$