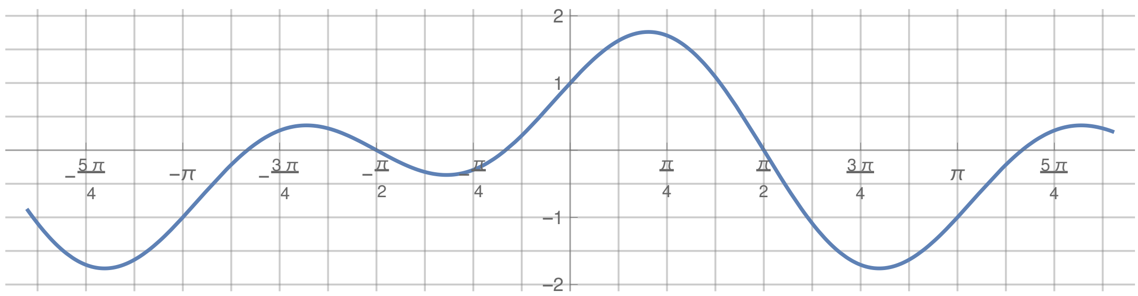
BITL3 # Worksheet 1 Math Lab

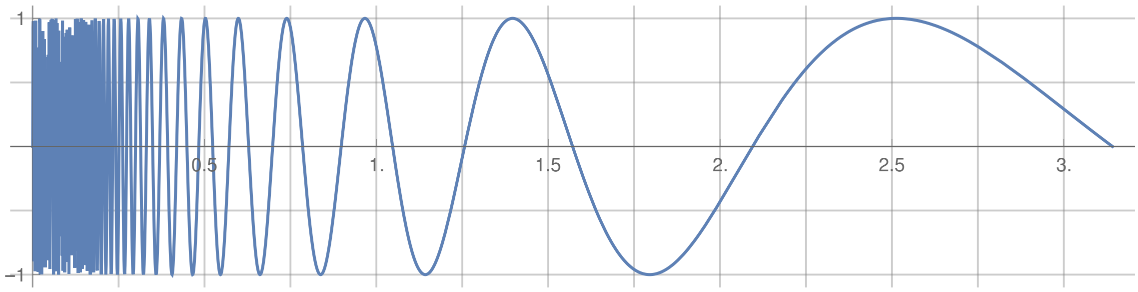
3 January 2022

1. **Warm up:** Answer the following True / False questions.
   (a) The statement "I had a good winter break" is true.
   (b) Either "I feel well rested" or "I did not think about math for the past two weeks" is true.
   (c) Both statements "a continuous function has a derivative" and "circles are not functions" are true.

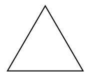
2. Recall that a root of a function f is approximated iteratively by the **Newton–Raphson method** by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, with x_0 the first guess and $x_{n\gg 0}$ a close approximation. A sequence $\{x_0, x_1, \ldots\}$ is produced by this method.
   (a) For each of the following, draw an example. Can you come up with a formula?
      i. a function f and a guess x_0 for which $f(x_0) \neq 0$ but $f(x_1) = 0$
      ii. a function g and a guess x_0 for which $f(x_n) \neq 0$ for all $n \in \mathbf{N}$ (the sequence is *divergent*)
      iii. a function h and a guess x_0 for which $x_{2n} = x_0$ and $x_{2n+1} = x_1$ for all $n \in \mathbf{N}$ (the sequence is *periodic*)
   (b) Below is the graph of $f(x) = \sin(2x) + \cos(x)$. Using a ruler, find 10 different points on f (and not on the x-axis) so that the tangent line passes through a zero of f. Are these points good or bad starting points?



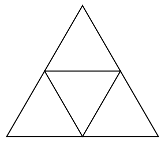
   (c) Below is the graph of $g(x) = \sin(2\pi^2/x)$. Using a calculator, compute $x_1, \ldots, x_5$ for $x_0 \in \{2.5, 2.49, 2.48, 2.47\}$. How similar is x_5 among the different choices of x_0? Can you conclude anything about the sequences $\{x_0, x_1, \ldots\}$, as in, will they converge or diverge?



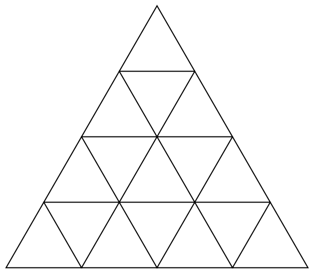
3. In this question we're going to construct triangles with **sequences**.
   Step 1: Begin by drawing a simple triangle, with three edges, as below.



   Step 2: Draw three of the triangles in the previous step to draw a new triangle, as below.



   Step 3: Draw three of the triangles in the previous step for a new triangle, and fill in any holes in the middle with more edges, as below.



   (a) Create a table for the step number and number of edges needed to draw the triangle.

step	1	2	3	4	5
edges needed	3	9			

   (b) For steps 1-5, give the number of edges needed in terms of the number of edges used for the previous step(s). For example, describe the number of edges needed for step 3 using the numbers 9 and 3 (the number of edges needed for the previous steps).

   (c) Can you describe how many edges will be needed for step n?

4. Use the **intermediate value theorem** (IVT) for the following questions.

   (a) What are the *conditions* and the *conclusion* of the IVT?

   (b) Show that $f(x) = x^3 + 4x - 1$ has a root between 0 and 1.

   (c) Does $g(x) = x + e^x$ have a root between -1 and 0? Why or why not?

   (d) Explain how to use the IVT to show that a car driving 120km in 1 hour must have driven faster than the speed limit of 90km/h at some point.
   Bonus: How long did the car have to be above the speed limit?