BITL3 | # Worksheet 24 | **Math Lab**

2 December 2021

Recall the following definitions for a continuous function $f\colon \mathbf{R} \to \mathbf{R}$. The function f has a

- **local minimum** at $a \in \mathbf{R}$ if $f(a) \leqslant f(x)$ for all x in a small neighbourhood around a
- **local maximum** at $a \in \mathbf{R}$ if $f(a) \geqslant f(x)$ for all x in a small neighbourhood around a
- **global minimum** on $I \subseteq \mathbf{R}$ at $a \in I$ if $f(a) \leqslant f(x)$ for all $x \in I$
- **global maximum** on $I \subseteq \mathbf{R}$ at $a \in I$ if $f(a) \geqslant f(x)$ for all $x \in I$

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1. **Warm up:** Answer the following questions.

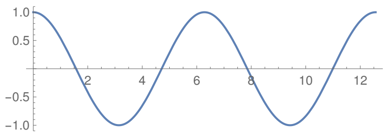
   (a) How many maxima can a continuous function with 2 local minima have?

   (b) True or False: A monotonic function can not have any local minima or local maxima.

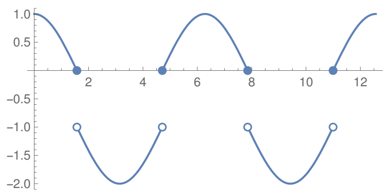
   (c) Which of the functions on the given intervals have which of the following: local min, local max, global min, global max?

   x^2 on $[-3, 5]$ $\qquad$ x^3 on $[-1, 10)$ $\qquad$ $\sin(x)$ on $(-\pi, \pi/4]$ $\qquad$ $\arctan(x)$ on $\mathbf{R}$

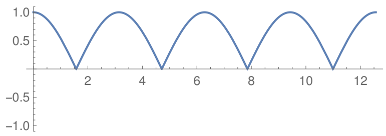
2. (a) For each of the functions below, draw its derivative over the function.



$f(x)$



$g(x)$



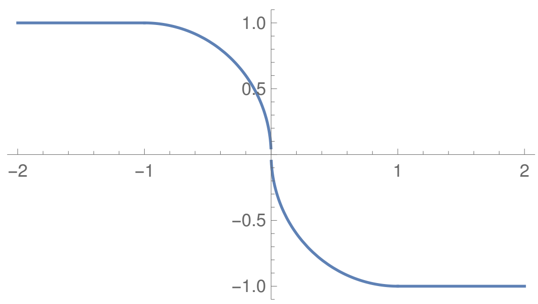
$h(x)$

   (b) The function g is not continuous at $x = \frac{\pi}{2} + k\pi$, for $k = 0, 1, 2, 3$. Is its derivative g' continuous at those points?

   (c) Draw the function $f(x) - g(x)$ on the interval $[0, 2\pi]$. What is its derivative?

3. Consider the function below and its graph.

$$f(x) = \begin{cases} 1 & \text{if } x < -1, \\ \sqrt{1-(x+1)^2} & \text{if } x \in [-1, 0), \\ -\sqrt{1-(x-1)^2} & \text{if } x \in [0, 1), \\ -1 & \text{if } x \geqslant 1. \end{cases}$$



   (a) Compute the first derivative $f'(x)$. Is is continuous at $x = -1, 0, 1$?

   (b) Compute the second derivative $f''(x)$. Is is continuous at $x = -1, 0, 1$?

4. Consider the functions $f_1(x) = |x-1|$ and $f_2(x) = |x+1| + 1$.

   (a) Graph $f_1(x)$, $f_2(x)$, and their sum $f(x) = f_1(x) + f_2(x)$.

   (b) Find all maxima and minima of f, and label them as local and/or global.

   (c) **Bonus:** Repeat the same process as in (b) for

$$g(x) = |x-3| - |x-2| + |x-1| + |x+1| - |x+2| + |x+3| + 1.$$

5. Consider the function $f(x) = \frac{6}{x^2+3}$.

   (a) Find where f reaches its largest and smallest values.

   (b) Find where the slopes of tangent lines of f are steepest (that is, have the largest positive values and the largest negative values).

   (c) **Bonus:** Do the same as in part (b), but for f'.