BITL3 **Math Lab**

Worksheet 23

1 December 2021

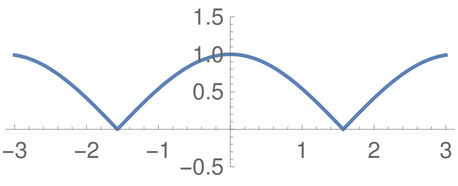
1. **Warm up:** Answer the following questions.

   (a) In you own words, what is a monotonic function? Draw a picture.

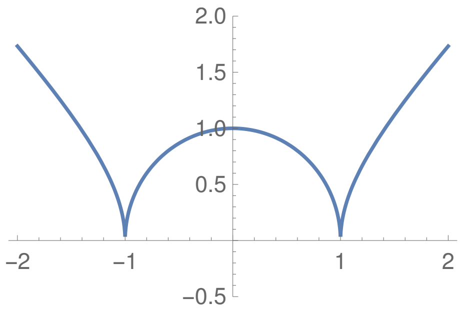
   (b) Without graphing the functions below, how many local maxima and local minima do they have on all of $\mathbf{R}$?

   $$x^2 \qquad x^{10} \qquad (x-4)(x-3)(x-2)(x-1)(x+1)(x+2)$$

   (c) What is the difference between the two "corners" on the x-axis of f and g?

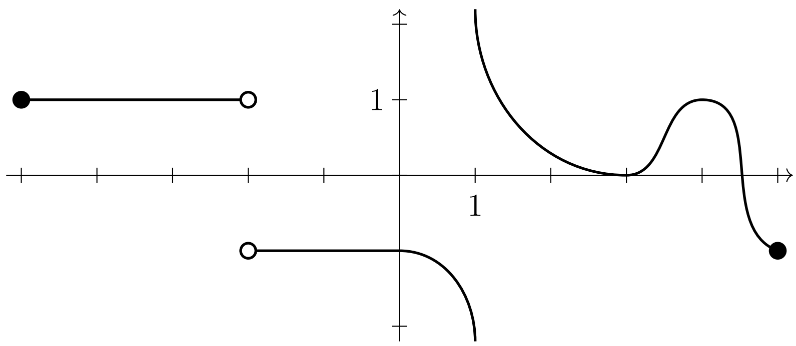


$f(x) = |\cos(x)|$



$g(x) = \sqrt{|1-x^2|}$

2. Below is the graph of the derivative f' of a continuous function f on $[-5, 5]$.



   (a) On what intervals is f increasing? Decreasing?

   (b) Where does f have critical points? Stationary points?

   (c) What are the x-values of the local minima and maxima of f?

   (d) On top of the graph above, draw a possible continuous function f that could have the graph as derivative.

   (e) On top of the graph above, draw the derivative f'' of f'.

3. How many maxima and minima do each of the functions have on the given interval? Find the coordinates (x, y) where these extrema occur.

   (a) $y = x^2$ on $(-\infty, \infty)$

   (b) $y = x(x-5)(x+5)$ on $[-6, 6]$

   (c) $y = \tan(x)$ on $[-\pi/2, \pi/2]$

   (d) $y = \sin(x)$ on $[0, 4\pi)$

   (e) $y = e^x$ on $[-100, 100]$

   (f) $y = \arctan(x)$ on $(0, \infty)$

4. (a) Find the smallest minimum of $f(x) = (x-1)^2 + (x-5)^2$ and its x-value.

   (b) Find the smallest minimum of $f(x) = (x-a)^2 + (x-b)^2$ and its x-value.

   (c) Find the smallest minimum of $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$ and its x-value.

   (d) What do you think is the x-value of the smallest minimum of $f(x) = \sum_{i=1}^{n} (x - a_i)^2$?

5. For each table below, draw a continuous function f and its derivative f' that satisfies the situation. The indicated x-values are critical points.

   (a)

$f'(x)$	$+$		$-$		$+$		$-$
x		-1		1		5	

   (b)

$f'(x)$	$+$		$-$		$-$		$+$
x		1		2		3	

   (c)

$f'(x)$	$-$		$-$		$+$		$+$		$+$
x		-1		0		10		20	

Bonus: Come up with a formula for each f.