BITL3 **Math Lab**

Worksheet 22

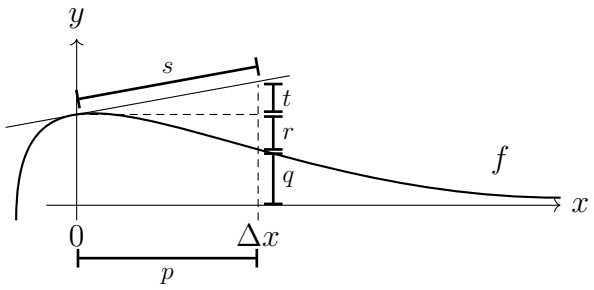
25 November 2021

1. **Warm up:** Answer the following questions.

   (a) For which of the following limits can l'Hôpital's rule be used:

   $$\lim_{x\to 2}\frac{\sin(\pi x)}{x-2} \qquad \lim_{x\to -9}\frac{x^2-7x-18}{\ln(|x|-9)^{-1}} \qquad \lim_{x\to 0} x^x$$

   (b) The following diagram is the graph of a function f and the tangent line at $x=0$. Which of the labeled values p, q, r, s, t is the *differential* for Δx at $a=0$?



   (c) Give an example of a function defined on $[0, 100]$ that is not linear, but is unit elastic on this interval.

2. Let $L_a(x)$ be the linear approximation to $\cos(x)$ at $x=a$.

   (a) At which value in $[0, \pi/2]$ are the differentials from $L_0(x)$ and $L_{\pi/2}(x)$ equal?

   (b) At which value in $[-\pi/2, \pi/2]$ are the differentials from $L_{\pi/2}(x)$ and $L_{-\pi/2}(x)$ equal?

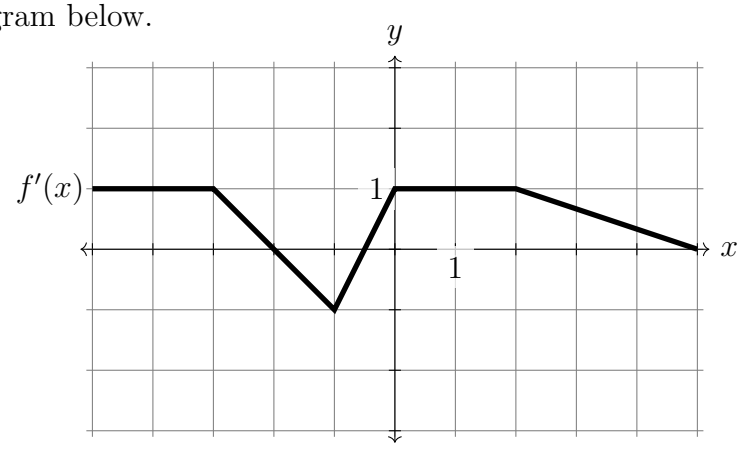
3. For each part of this question, decribe a continuous function. Find a function f, using $a=0$, for which:

   (a) the differential for Δx is 1

   (b) the differential for $k\cdot\Delta x$ is k, for all $k\in\mathbf{N}$

   (c) the differential for $k\cdot\Delta x$ is k^2+2k-1 for all $k\in\mathbf{R}$, $k\geqslant 1$

4. Let f be a continuous function defined on $[-5, 5]$. The graph of f', the derivative of f, is given in the diagram below.



   (a) Draw one possible f on the same diagram.

   (b) Find the points (x, y) on the graph of f where its tangent line is horizontal.