BITL3 | **Math Lab**

Worksheet 21

24 November 2021

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l'Hopital's rule states that if $\lim_{x\to a} f(x) = \pm \lim_{x\to a} g(x) = L$, and $g(a) \neq 0$, then

$$L = 0 \text{ or } L = \infty \qquad \Longrightarrow \qquad \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Both functions have to be differentiable in a neighborhood of a (but not necessarily at a).

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1. **Warm up:** Answer the following questions.
   (a) What is the slope of the line perpendicular to the tangent line of $y = x^2$?
   (b) Explain in what cases can l'Hopital's rule be applied to $\lim_{x\to a} f(x)^{g(x)}$.
   (c) True or False: The closer a linear approximation of f is taken to a particular value, the better estimate it will have of f of that value.

2. Let L_0 be the linear approximation of x^2 at 0, and L_k the linear approximation of x^2 at (k, k^2). Find the point (x, y) where L_0 intersects L_k.

3. Evaluate the following limits using l'Hôpital's rule and other differentiation rules you know.
   (a) $\lim_{x\to 1} \frac{x^n - 1}{x - 1}$
   (b) $\lim_{z\to 0} \frac{\tan(4z)}{\tan(7z)}$
   (c) $\lim_{x\to 2^+} \frac{1}{x-2} - \frac{1}{\ln(x-1)}$
   (d) $\lim_{x\to 0^+} (e^x + x)^{\frac{1}{4x}}$
   (e) $\lim_{x\to 0^+} \frac{\sin(x) - x}{x^3}$
   (f) $\lim_{x\to 0^+} (\cos(x) - 1)^x$
   (g) $\lim_{x\to \pi/2^+} \frac{\sec(x)}{1 + \tan(x)}$
   (h) $\lim_{x\to 0^+} \frac{2\ln(e^x - 1)}{\ln(3x)}$

4. Suppose that at price p, for $p \in (0, 10)$, the demand for a product is $f(p)$ kilograms, where $f(p) = 120 - 2p - p^2$.
   (a) What is the price elasticity of demand when $p = 5$?
   (b) What is the average elasticity of demand in the price interval $[5, 7]$?
   (c) Is demand for this product elastic or inelastic on the domain $(0, 10)$? Why?