Worksheet 20

- 1. Warm up: Answer the following True / False questions.
 - (a) If f(x) is constant, then f(g(x)) is also constant.
 - (b) If f'(x) and g'(x) are positive, then $\frac{d}{dx}(f(g(x)))$ is positive.
 - (c) An implicitly defined function can always be expressed by two explicitly defined functions.
- 2. **Elasticity** is a measure of how a variable changes when another variable that it depends on changes.
 - (a) The *force* of gravity depends on the mass of the bodies involved and the distance between them: $F(m_1, m_2, d) = 6.67 \cdot 10^{-11} \cdot m_1 \cdot m_2 \cdot d^{-2}$.
 - i. Compute the distance elasticity of force, when distance is 10 and 10^{10} .
 - ii. Compute the mass m_1 elasticity of force, when mass m_1 is 10 and 10^{10} .
 - (b) The *demand* of an electric car depends on the *price* of electric batteries that are used in them: $D(p) = 10p(p-20)(p-30) + 10^4$.
 - i. Compute the price elasticity of demand when price is 10 and 40.
 - ii. For what prices p will demand be unit elastic?
- 3. Consider the implicitly defined function $y^2(8y^2 31) = (2x + 1)((2x + 1)^2 24)$, whose graph is given below.



- (a) Compute the derivative $\frac{dy}{dx}$.
- (b) Where does the line x = 0 intersect the graph? That is, find all points (x = 0, y) that satisfy the implicit equation.
- (c) Find the slopes of the tangent lines from the points found in part (b).
- (d) Find the equations of the tangent lines from two of the points found in part (b).
- 4. Consider the function $f(x) = e^{-x^2 + \ln(x)}$.
 - (a) Find the derivative f'(x).
 - (b) Find the points (x, y) on the graph of f where the tangent line to f is horizontal.
 - (c) Where is the derivative positive? Where is it negative?