Worksheet 19

11 November 2021

1. **Warm up:** Compute the derivative with respect to x of the following functions.
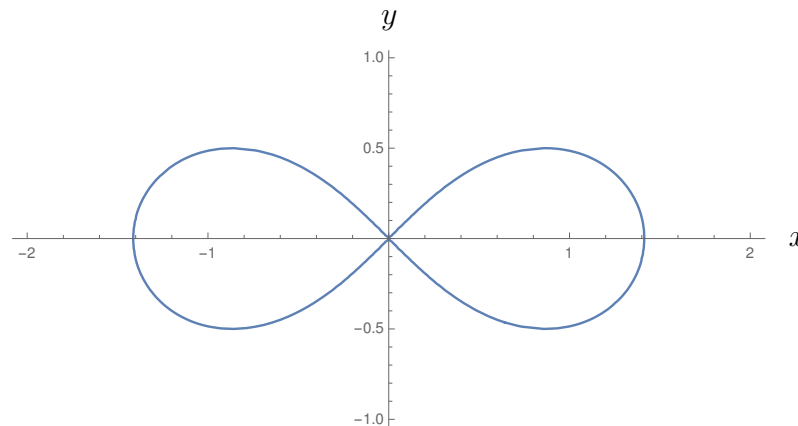
   (a) x^x

   (b) $(x^x)^x$

   (c) $x^{(x^x)}$

   (d) $(x^x)^{(x^x)}$

2. Below is the graph of $(x^2 + y^2)^2 = 2x^2 - 2y^2$.

   Just by looking at the graph, answer the following questions.

   (a) How many points on the graph are there for which $\dfrac{dy}{dx} = 0$?

   (b) Choose any real number c.

      i. How many points on the graph are there for which $\dfrac{dy}{dx} = c$?

      ii. How many points on the graph are there for which $\dfrac{dx}{dy} = c$?

3. For each relationship below, find the equation of the tangent line to the curve at the given point.

   (a) $x^3 + xy + y^2 = 7$ at $(2, 1)$
   (b) $(x + y)^{2/3} = y$ at $(4, 4)$

4. Consider the implicitly defined relationship $y = x^2 y^3 + x^3 y^2$.

   (a) What values of y satisfy the relationship when $x = 1$?
   (b) Compute $\frac{dy}{dx}$ at $x = 1$.
   (c) What values of x will satisfy $\frac{dx}{dy} = -1$ at $y = 1$?

5. A spherical balloon is inflated and its volume increases at a rate of $15cm^3/min$. What is the rate of change of its radius, per minute, when the radius is $10cm$?