BITL3

# Worksheet 18

10 November 2021

**Math Lab**

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Recall the following rules for differentiation:

- **product rule:** $\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$
- **quotient rule:** $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- **chain rule**: $\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))g'(x)$

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1. **Warm up:** Answer the following True / False questions.
   (a) If a function is differentiable at a point $a$, then it is continuous at $a$.
   (b) If a function is continuous at a point $a$, then it is differentiable at $a$.
   (c) If $f(x) = 3x^2 - 2$, then $f'(5) = f'(3) + f'(2)$.
   (d) There is no difference betwen $\frac{d}{dx}(5x^2 + 2xy - 3y^2)$ and $\frac{d}{dy}(5x^2 + 2xy - 3y^2)$.

2. Differentiate the following functions, with respect to $z$.
   (a) $\sqrt{z+2}\ln(z^3 + 2z)$
   (b) $\ln(\arctan(3^z)/\pi) + 22$
   (c) $2\cos(z)^{z+1}$
   (d) $10z^2 + (e^{z+3}/e^3)^{z^2}$

3. Suppose $f$ is a differentiable function with the following value for $f$ and $f'$ as given below.

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 0 | 3 | $-1$ |
| 1 | 5 | 0 |
| 2 | $-2$ | 3 |
| 3 | 6 | 1 |

   Let $g(x) = x^2 - 3x + 2$. For each function below, calculate its derivative at the given point.
   (a) $f(x) + g(x)$ at $x = 0$
   (b) $\frac{f(x)}{g(x)}$ at $x = 1$
   (c) $f(x)g(x)$ at $x = 2$
   (d) $\frac{f(x)g(x)}{f(x) + g(x)}$ at $x = 3$
   (e) $f(g(x))$ at $x = 0$
   (f) $f(g(x))$ at $x = 1$
   (g) $g(f(x))$ at $x = 2$
   (h) $g(f(x))$ at $x = 3$

4. The sine and cosine functions may be defined using the **imaginary number** $i$. The number $i$ is not a real number, and is defined by its square $i^2 = -1$. The formulas are

$$\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}, \qquad \cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}.$$

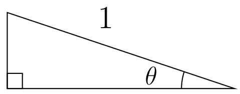
(a) Using these formulas, show that $\frac{d}{d\theta}\sin(\theta) = \cos(\theta)$ and $\frac{d}{d\theta}\cos(\theta) = -\sin(\theta)$.

(b) Using the sum and difference formula for $\sin(a \pm b)$ and $\cos(a \pm b)$, express $\sin(x + iy)$ and $\cos(x + iy)$ in terms of the exponential function.

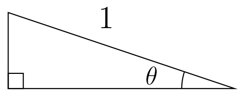
5. Recall the following functions and their derivatives:

| $f(x)$ | $\sin(x)$ | $\cos(x)$ | $\tan(x)$ | $\csc(x)$ | $\sec(x)$ | $\cot(x)$ |
|---|---|---|---|---|---|---|
| $f'(x)$ | $\cos(x)$ | $-\sin(x)$ | $\sec^2(x)$ | $-\cot(x)\csc(x)$ | $\sec(x)\tan(x)$ | $-\csc^2(x)$ |

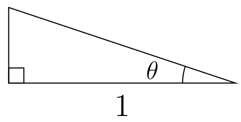
(a) For each of the triangles below and the condition given, find the missing side lengths.




$x = \sin(\theta)$




$x = \cos(\theta)$




$x = \tan(\theta)$

(b) Use the chain rule and the definitions of the inverse trigonometric functions, like $\sin(\arcsin(x)) = x$, to complete the table of derivatives below.

| $f(x)$ | $\arcsin(x)$ | $\arccos(x)$ | $\arctan(x)$ | $\text{arccsc}(x)$ | $\text{arcsec}(x)$ | $\text{arccot}(x)$ |
|---|---|---|---|---|---|---|
| $f'(x)$ | | | | | | |