

There are two important limits related to the **exponential number** e :

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e, \quad \text{and} \quad \lim_{x \rightarrow \infty} \left(1 + \frac{1}{f(x)}\right)^{f(x)} = e,$$

for any function $f(x)$ with $\lim_{x \rightarrow \infty} f(x) \rightarrow \infty$.

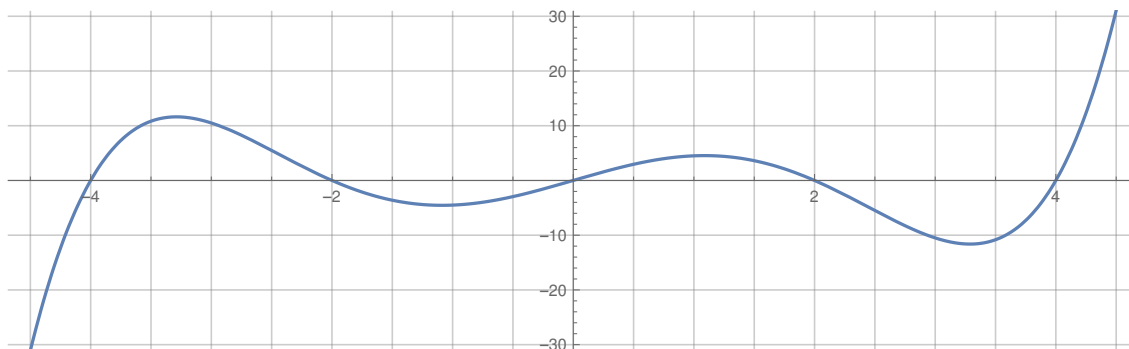
1. **Warm up:** What are the first and second derivatives of the following functions?

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|-------------------|-----------------------|---------------------|
| (a) $x + 2$ | (c) 2^x | (e) x^{-2} |
| (b) $x^2 \sin(x)$ | (d) $(x^{1/2})^{1/2}$ | (f) $e^{2x}/2^{ex}$ |

2. Use the exponential limit to evaluate the following limits.

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| (a) $\lim_{t \rightarrow 0} (1+t)^{1/t}$ | (c) $\lim_{x \rightarrow \infty} \left(\frac{2x-3}{2x+1}\right)^{\frac{x+1}{3}}$ |
| (b) $\lim_{n \rightarrow \infty} \left(1 + \frac{6}{2-n}\right)^{n+1}$ | (d) $\lim_{n \rightarrow -\infty} \left(\frac{n}{n-1}\right)^{2n}$ |

3. Below is the graph of $f(x) = \frac{x^5}{10} - 2x^3 + \frac{32x}{5} = \frac{x}{10}(x+2)(x-2)(x+4)(x-4)$.



- (a) Compute the derivative $f'(x)$.
- (b) At what x -values will the tangent line to f be horizontal?
- (c) What will be the y -values at the given x -values from part (b)?
4. Draw the graph of a continuous function f that satisfies the following conditions:
- $f'(0) \neq 0$
 - $f'(-1) = f'(1) = 0$
 - $f'(-2) = f'(2) = 1$
 - $f'(x) < 0$ when $x \leq -3$ and when $x \geq 3$
5. Let $f(x) = 2x^2 + 5$ and $g(x) = 4e^x - 1$.
- (a) What is the function $f(g(x))$? What is $g(f(x))$?
- (b) Compute the derivative $f'(x)$ and $g'(x)$.
- (c) What is $f'(g(x))$? What is $g'(f(x))$?