BITL3 **Math Lab**

Worksheet 16

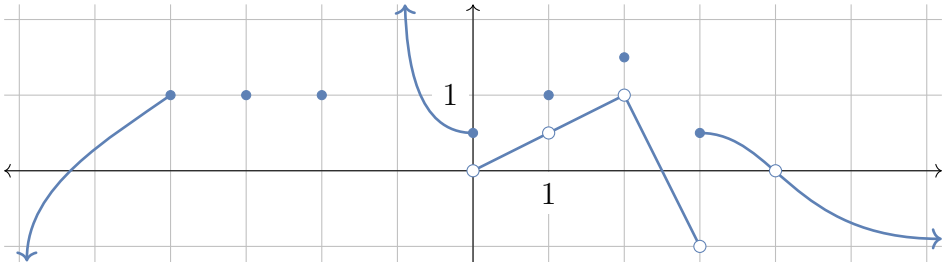
3 November 2021

The **equation of the tangent line** to the function f at the point $(a, f(a))$ is

$$y = f'(a)(x - a) + f(a).$$

1. **Warm up 1:** Answer these questions with "True" or "False".
   (a) If a line is tangent to a graph at a point, it only touces the graph at that point.
   (b) The exponential function e^x has two different points with equal tangent lines.
   (c) Given any line, there is always a function with that line as a tangent line at $x = 0$.

2. **Warm up 2:** Consider the graph of the function f given below.

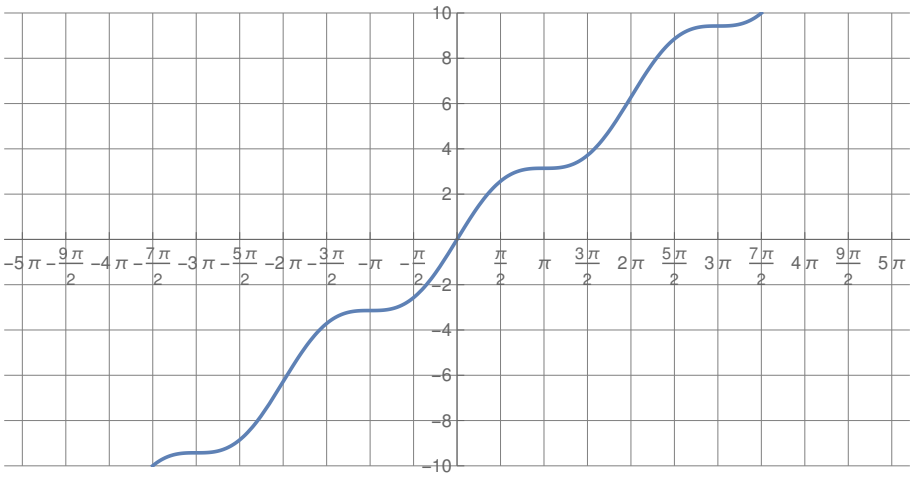


   (a) What is the domain of f?
   (b) What is the range of f?
   (c) What are the horizontal and vertical asymptotes of f?
   (d) At how many points on the domain of f is the function not continuous?

3. (a) Consider a circle C of radius r.
      i. What is the circumfrence of C?
      ii. What is the area of C?
      iii. What is the derivative of the area of C, with respect to r?
   (b) Consider a sphere S of radius r.
      i. What is the surface area of S?
      ii. What is the volume of S?
      iii. What is the derivative of the volume of S, with respect to r?

4. Let $f(x) = x^2 + cx + 1$.
   (a) For what values of c will f have two roots? One root? no roots?
   (b) For what value of c will the slope of f at $x = a$ be equal to a?
   (c) Find the equation of the tangent line to f at $x = 0$.

5. Below is the graph of the function $f(x) = \sin(x) + x$.



(a) For which x-values is the tangent line to f horizontal?

(b) Find two x-values x_1, x_2 at which the tangent lines to f are exactly the same. Guess from the graph, and confirm your guess by using the fact that $f'(x) = \cos(x) + 1$.

(c) What are the possible values of the slope of the tangent line to f?

6. Find an example of a function that has:

(a) tangent line $y = x + 2$ at $x = -1$ and $y = 3x + 4$ at $x = 1$

(b) tangent line $y = ax + b$ at $x = -1$ and $y = cx + d$ at $x = 1$

(c) tangent line $y = ax + b$ at $x = -2$ and $y = cx + d$ at $x = 2$, and which is continuous

The functions for parts (a) and (b) do not have to be continuous.