Worksheet 14

21 October 2021

Recall the **derivative** of a function f at a point $a \in D_f$ is

$$\underbrace{f'(a) = \frac{df}{dx}(a) = \frac{df}{dx}\Big|_{x=a}}_{\text{different ways to write the same thing}} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

A function f is **continuous at** $a \in D_f$ if $\lim_{x \to a} f(x)$ exists and is equal to $f(a)$.

A function f is **differentiable at** $a \in D_f$ if $f'(a)$ exists.

If two functions f, g are differentiable at the same points, then, for $c \in \mathbf{R}$:

- the derivative of the sum is the sum of the derivatives: $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

- the derivative of a multiple is a multiple of the derivative: $(c \cdot f(x))' = c \cdot f'(x)$

- the derivative of a power function is a power function: $\dfrac{d}{dx}(x^c) = c \cdot x^{c-1}$

1. **Warm up:** Answer the following True / False questions.

    (a) If f is not defined at a (so $a \notin D_f$), then f is not differentiable at a.

    (b) If f is continuous at $a \in D_f$, then f is differentiable at a.

    (c) If f is differentiable at $a \in D_f$, then f is continuous at a.

2. Below are six graphs.

    | 1 | 2 | 3 | 4 | 5 | 6 |
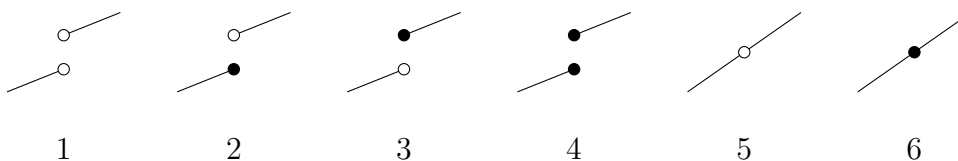
    Which of these graphs satisfy the properties below?

    (a) the graph is a function

    (b) the graph is defined at every point

    (c) left-side limits exist at every point

    (d) right-side limits exist at every point

    (e) limits exist at every point

    (f) the graph is a function f and $\lim_{x \to a^-} f(x) = f(a)$ for all points a

    (g) the graph is a function f and $\lim_{x \to a^+} f(x) = f(a)$ for all points a

3. (a) Use the limit definition to compute $f'(a)$, the derivative at a point.

    i. $f(x) = x^2 - 2x + 3$, $a = 1$ ii. $f(x) = \frac{1}{x+1}$, $a = 3$

(b) Use the derivative laws to compute $f'(x)$, the derivative function.

    i. $f(x) = x^2 - 2x + 3$ ii. $f(x) = \frac{2x^{22} - 0.2 \cdot x^{0.22}}{22}$

4. Let $f(x) = x^4 - 4x^3 + 4x^2 + 4$.

   (a) Compute the derivative function $f'(x)$.

   (b) Show that there are exactly three x-values at which the slope of f equals 0.

   (c) Find the coordinates (x, y) of these three points.