BITL3 **Math Lab**

Worksheet 12

14 October 2021

Recall the **instantaneous rate of change** of a function f at a point a in its domain is

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

This is a **limit**. The **average** rate of change of f over the interval $[a,b]$ is

$$\frac{f(b)-f(a)}{b-a}=\frac{\Delta y}{\Delta x}.$$

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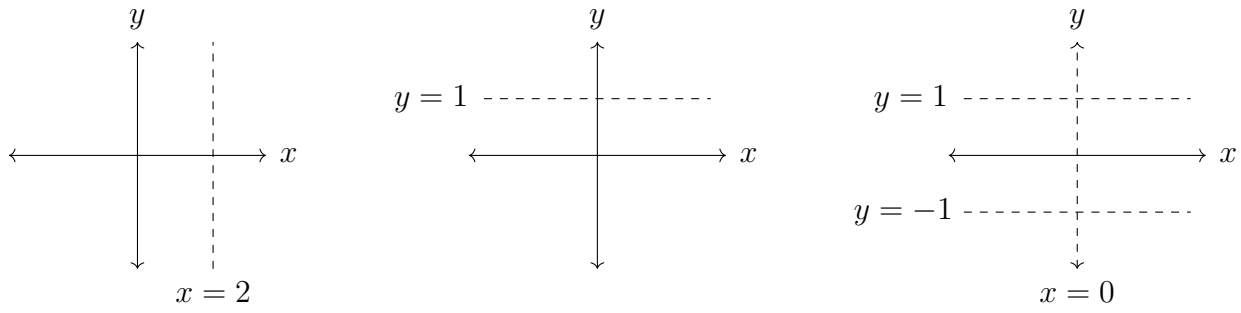
1. **Warm up:** Fill in the blank to the following questions with one of the answers *0, 1, 2, finitely many*, or *infinitely many*.
   (a) A function can have at most _____ vertical asymptotes.
   (b) A function can have at most _____ horizontal asymptotes.
   (c) A function can have at most _____ zeros.

2. (a) Draw on the axes below graphs of three functions that have only the indicated asymptotes, and no other asymptotes.



   (b) Give formulas for each of the graphs above.

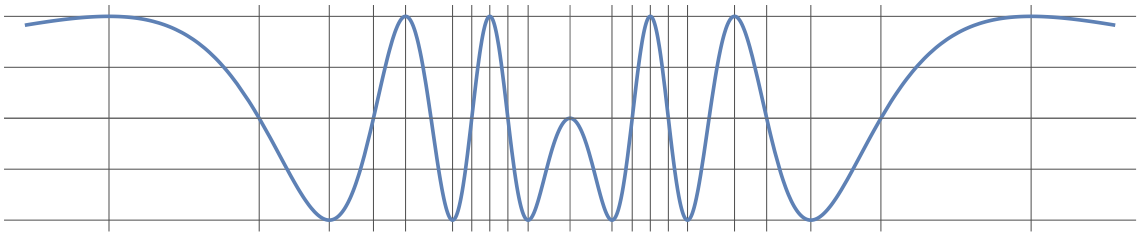
3. Consider the piecewise function

$$h(x)=\begin{cases}x^2+1 & \text{if } x<0,\\ \cos(x) & \text{if } 0\leqslant x<2\pi,\\ \frac{1}{x-3\pi} & \text{if } x\geqslant 2\pi.\end{cases}$$

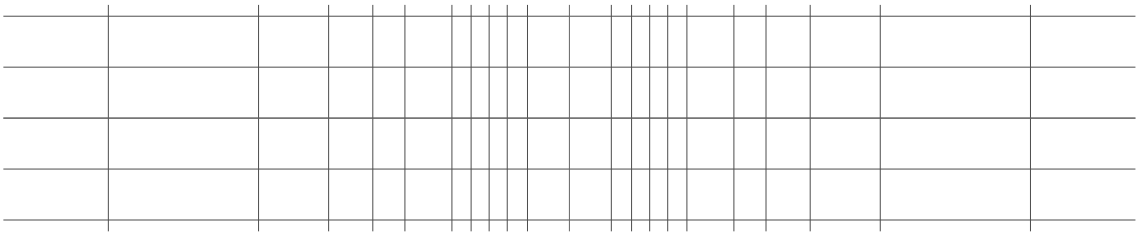
   (a) Where is the tangent line to h horizontal?
   (b) Where is the tangent line to h vertical?
   (c) At which points is the tangent line to h not defined?

4. Consider the function $f(x)=\dfrac{|x|-|x-2|}{x-1}$.
   (a) Rewrite f as a piecewise function, split on the intervals $(-\infty,0)$, $[0,2)$, $[2,\infty)$.
   (b) Draw f on the interval $[-3,3]$.
   (c) Evaluate $\lim_{x\to 1} f(x)$.

5. Consider the function f below.



(a) On the grid with the same background below, indicate the points (x, slope at $f(x)$), for all spots where the tangent line is horizontal.



(b) On the same plot, indicate the points (x, slope at $f(x)$) for all spots where the graph of f intersects the x-axis.

(c) Connect the lines to estimate what the "slope plot" looks like.