BITL3

Worksheet 11

13 October 2021

Math Lab

1. **Warm up:** Answer the following True / False questions.

   (a) If $\lim_{x\to 0} f(x) = 1$, then $\lim_{x\to 0}(f(x) - 1) = 0$.

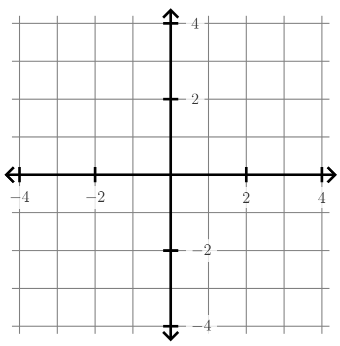
   (b) For any two functions f and g, $\lim_{x\to 5} f(x) + \lim_{x\to 5} g(x) = \lim_{x\to 5} f(x) + g(x)$.

   (c) Every function f has a tangent line at $f(x)$, for all $x \in D_f$.

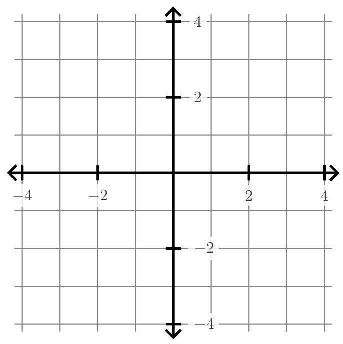
   (d) As long as $\lim_{x\to 0} f(x)$ exists, $\lim_{x\to 0} 2f(x) \geqslant \lim_{x\to 0} f(x)$.

2. The *floor function* $\lfloor x \rfloor$ gives to every real number x the value of the largest integer less than or equal to x. The *ceiling funcion* $\lceil x \rceil$ gives to every real number x the value of the smallest integer larger than or equal to x.

   (a) Draw the graph of $\lfloor x \rfloor$.



   (c) Draw the graph of $x - \lfloor x \rfloor$.

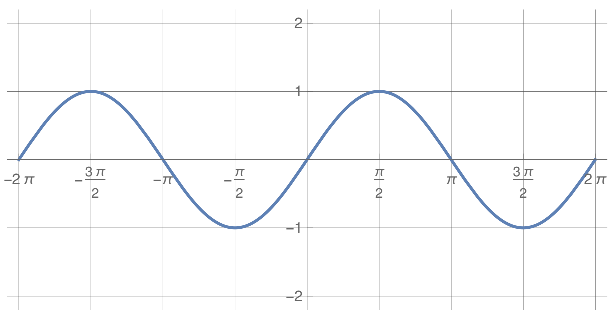


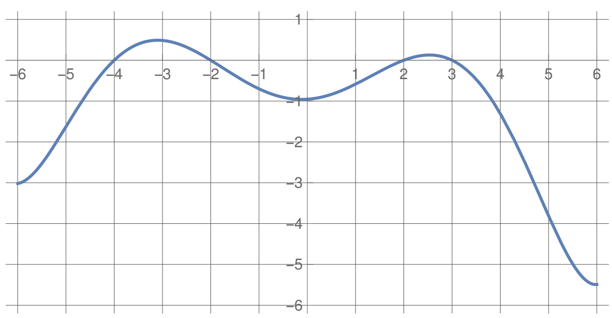
   (b) What is the range?

   (d) What is the range?

   (e) Do these functions have inverses everywhere / nowhere / on a subset of the domain?

   (f) Below are two functions f and g. Draw the graphs $\lfloor f(x) \rfloor$ and $\lfloor g(x) \rfloor$ over them.





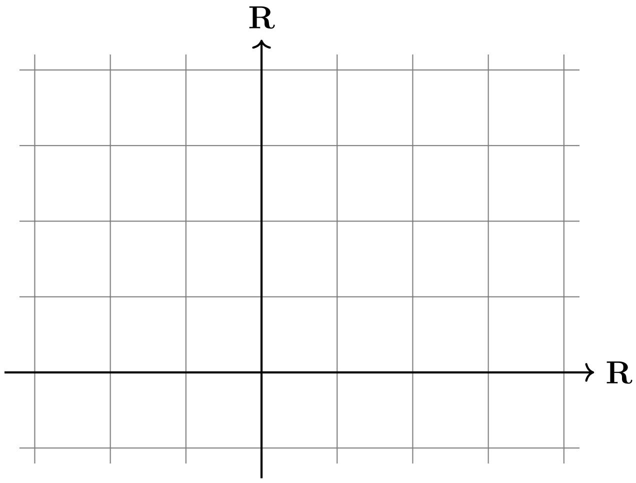
   (g) **Bonus:** Find an equation that relates $\lfloor x \rfloor$ and $\lceil x \rceil$.

3. Consider the piecewise function:

$$f(x) = \begin{cases} x^2 & \text{if } x \leqslant -1, \\ ax+b & \text{if } -1 < x < 2, \\ |x| & \text{if } x \geqslant 2. \end{cases}$$

(a) Find a pair of real numbers a, b that make the function f have no jumps.

(b) For the a, b you found, graph the function on the grid below.



4. Compute the average rate of change of the function $f(x) = 45\sin(x)^2 + 22x^3 - 4$ on the interval $[0, \pi]$.

5. Find a general expression for the average rate of change of the function $f(x) = x(x-a)$ on the interval $[a-h, a]$.