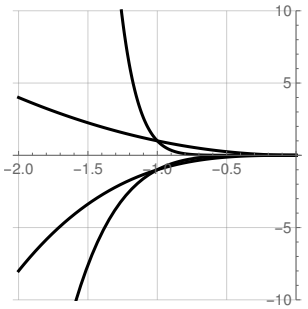
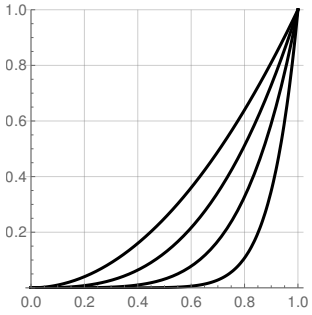
BITL3 **Math Lab**

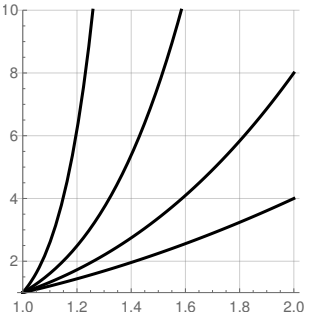
Worksheet 8

30 September 2021

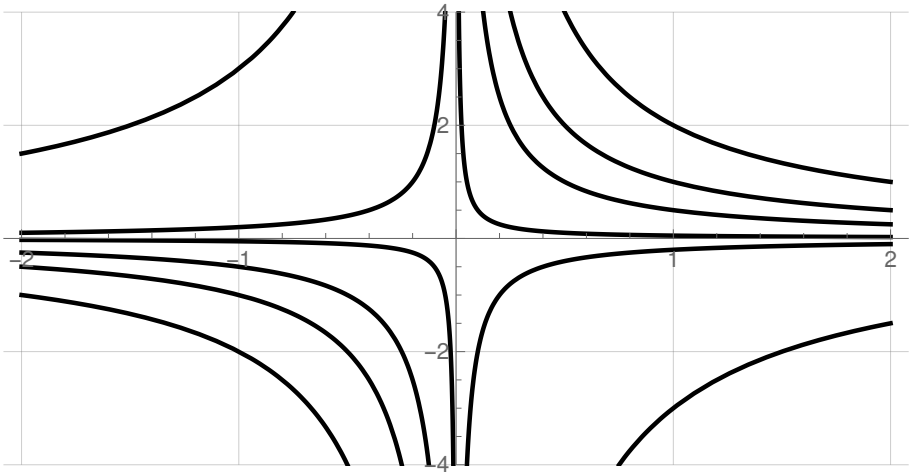
1. **Warm up 1:** Four power functions x^a, x^b, x^c, x^d are plotted below, with integer powers. You are given that $a < b < c < d$, and that a, d are even and b, c are odd. Identify the functions in each of the plots.







2. **Warm up 2:** Five inverse functions a/x, b/x, c/x, d/x, e/x, f/x are plotted below, with real numbers a, b, c, d, e, f. You are given that $a < b < c < d < e < f$. Identify the functions in each of the plots. Be sure to indicate which "pieces" belong to the same graph. Which of the values are positive and which are negative?



3. Solve the following problems by induction.

   (a) $10^n - 1$ is divisible by 3 for all $n \in \mathbf{N}$

   (b) $\displaystyle\sum_{i=1}^{n} 3i^2 = \frac{n(1+n)^2 + n^2(1+n)}{2}$ for all $n \in \mathbf{N}$

   (c) The relationship $0 \leqslant f(n) \leqslant f(n+1) \leqslant 3$ is true for all $n \in \mathbf{N}$, given that

$$f(1) = 1, \qquad f(n+1) = \sqrt{3 + 2f(n)} \quad \text{for } n \geqslant 1.$$