BITL3 **Math Lab**

Worksheet 7

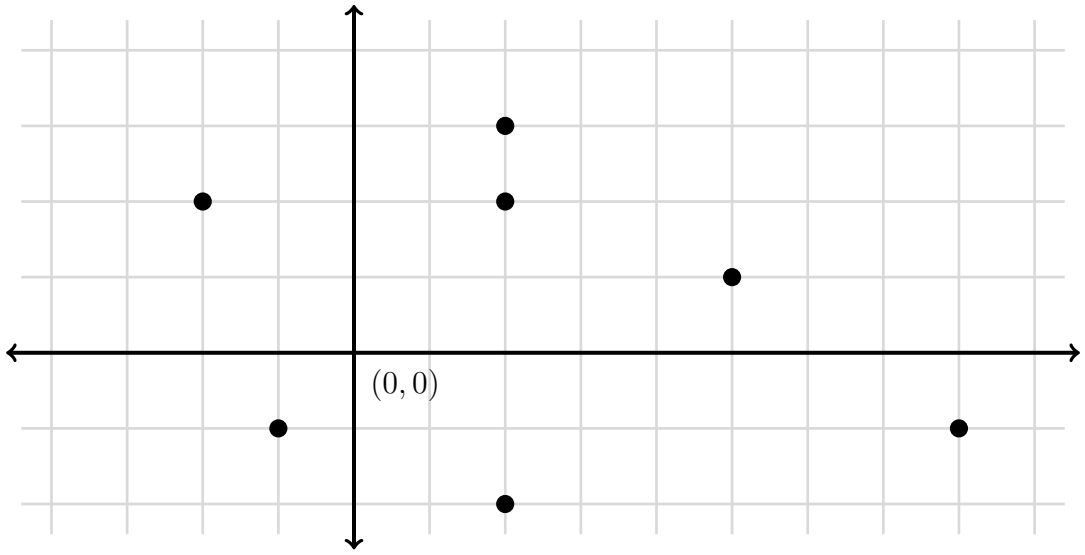
29 September 2021

Recall the following definitions. For all $x \in \mathbf{R}$, function $f\colon \mathbf{R} \to \mathbf{R}$ is

- **increasing** if $f(x) < f(x+c)$ for all $c \in \mathbf{R}_{>0}$
- **decreasing** if $f(x) > f(x+c)$ for all $c \in \mathbf{R}_{>0}$
- **constant** if $f(x) = c$ for some $c \in \mathbf{R}$
- **odd** if $f(x) = -f(-x)$
- **even** if $f(x) = f(-x)$
- **periodic** if $f(x) = f(x+p)$ for some $p \in \mathbf{R}$, called the **period**

1. **Warm up:** Answer the following True / False questions.
   (a) Every constant function is even.
   (b) Every constant function is periodic.
   (c) If a function is even, it can not be periodic.
   (d) If a function is increasing, it can not be odd.

2. Consider these 7 points in the plane.



   (a) How many linear functions can be drawn that pass through at least two points?
   (b) How many of them are increasing and how many are decreasing?
   (c) Determine the equations for 3 of the lines from part (a), assuming that the grid has squares of side length 1.
   (d) Draw an even function that is not linear through two of the points.
   (e) Draw an odd function that is not linear through two of the points.
   (f) **Bonus 1:** Draw a periodic function that is not linear through three of the points.
   (g) **Bonus 2:** How many quadratic functions can be drawn that pass through at least 3 points?

3. For each of the following functions, find their domains.

$$\ln(x) \qquad \ln(|x|) \qquad \sin(x) \qquad \sin(\ln(x)) \qquad \ln(\sin(x)) \qquad \sqrt{\ln(\sin(x))}$$

4. Let f and g be functions. Complete the table below by determining if the functions $f+g$ and $f \cdot g$ are odd, even, constant, periodic, or without any symmetries.

f	g	$f+g$	$f \cdot g$
even	even		
odd	odd		
odd	even		
even	constant		
even	periodic		

5. Prove the following statements by induction.

   (a) $(1+x)^n \geqslant 1+nx$ for all $n \in \mathbf{N}$ and $x \in \mathbf{R}_{>-1}$.

   (b) $\dfrac{x^n-1}{x-1} = 1+x+x^2+\cdots+x^{n-1}$ for all $n \in \mathbf{N}$.