BITL3 | **Math Lab**

Worksheet 3

15 September 2021

In this worksheet we will use the following definitions.

- A **polynomial** is a finite sum of real numbers a_n multiplying different powers of variables x^n. For example, $3x + 2$ and $9x^2 - 10x^{10}$ are polynomials.
- The **degree** of a polynomial $f(x)$ is the largest power of x appearing in the sum, whose real number coefficient is not zero. It is denoted $\deg(f)$. For example, the degree of $4x^3 - 1$ is 3, even though this may be written as $4x^3 - 1 + 0 \cdot x^5$.
- A real number r is a **root** or **zero** of a polynomial $f(x)$ if $f(r) = 0$. Not all polynomials have roots. Polynomials can have at most $\deg(f)$ roots.

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1. **Warm up:** Indicate which of the following functions are polynomials and which are not.

$$f(x) = \frac{1}{2}x^2 + \pi x + 2$$

$$g(x) = 3x^2 - x^{2/3}$$

$$h(x) = x^2 + 2^x$$

$$k(x) = 5x^2 + 4x + 3 + 2x^{-1}$$

$$\ell(x) = 0$$

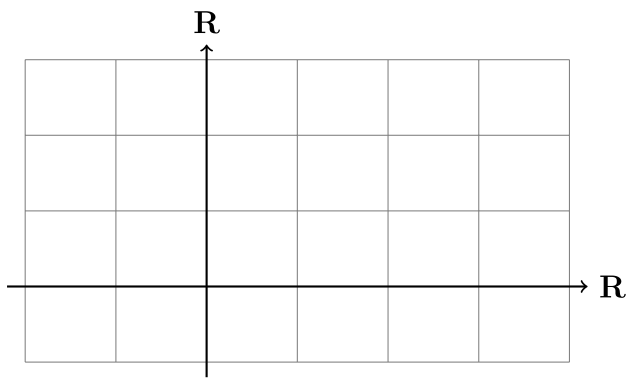
$$m(x) = \sum_{n=1}^{1000} nx^n$$

$$p(x) = \sum_{n=-1000}^{1000} nx^n$$

$$q(x) = \sum_{n=1000}^{\infty} nx^n$$

2. This question is about solving inequalities.
   (a) For what values of x is the inequality $x^2 + x - 2 > 0$ true?
   (b) Construct a quadratic function $f(x)$ so that the solution to $f(x) < 0$ is $x \in (-5, -1)$.
   (c) Find another quadratic function $g(x)$ for which the solution to $g(x) < 0$ is the same as in part (b). How many different functions with this same solution do there exist?
   (d) Suppose that that the solution to $f(x) \geqslant 0$ is $x \in [0, 3]$ and the solution to $g(x) \geqslant 0$ is $x \in [2, 4]$. What is the solution to $f(x)g(x) \geqslant 0$?

3. This question is about absolute value inequalities.
   (a) Draw the functions $y = |x|$ and $y = |x - 2|$ on the grid below.
   (b) Shade in the area where both $y \geqslant |x|$ and $y \geqslant |x - 2|$ are satisfied. Find a function $f(x)$ so that $y \geqslant f(x)$ has this same area as it solution set.



4. This question is about polynomial division.

   (a) Use polynomial division to simplify the expression $\dfrac{10x^3 - 3x + 15}{3x - 2}$.

   (b) Find the roots of $x^4 + 2x^3 - 25x^2 - 26x + 120$ with polynomial division. You may assume that the roots are integers in the set $\{-5, \ldots, 5\}$,