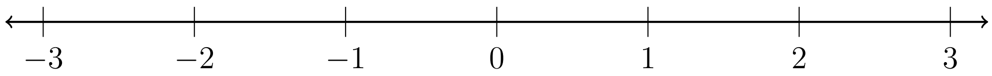
BITL3 | Math Lab

Worksheet 1

9 September 2021

In this worksheet we will use the following definitions.

- A **natural number** is an element of the ordered set $\mathbf{N} = \{1, 2, 3, 4, \dots\}$.
- An **integer** is an element of the set $\mathbf{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$. Another way to express this set is $\mathbf{Z} = \mathbf{N} \cup \{0\} \cup \{-n \ : \ n \in \mathbf{N}\}$.
- A **rational number** is an expression $\frac{a}{b}$, where $a, b \in \mathbf{Z}$ and $b \neq 0$. Every rational number has a **unique** way to write it, where $b > 0$ and there are no common factors among a, b.
- A **real number** is difficult to define. It is any number on the real line:

  Or, it is any number than can be approximated **arbitrarily close** by a rational number. You will learn more about this in the topic of **limits**. The set of real numbers is $\mathbf{R}$.
- An **irrational number** is a real number that is not rational. That is, $\mathbf{I} = \overline{\mathbf{Q}} = \mathbf{R} \setminus \mathbf{Q}$.

All of these sets are **ordered** by the relation $<$. For example, $\mathbf{Z}$ is ordered by $<$ in the way that $1 < 2$, $-52 < 10$, and so on. They are also all **closed** under addition and multiplication. For example, given two elements $a, b \in \mathbf{Z}$, their product ab and their sum $a+b$ are still elements of $\mathbf{Z}$.

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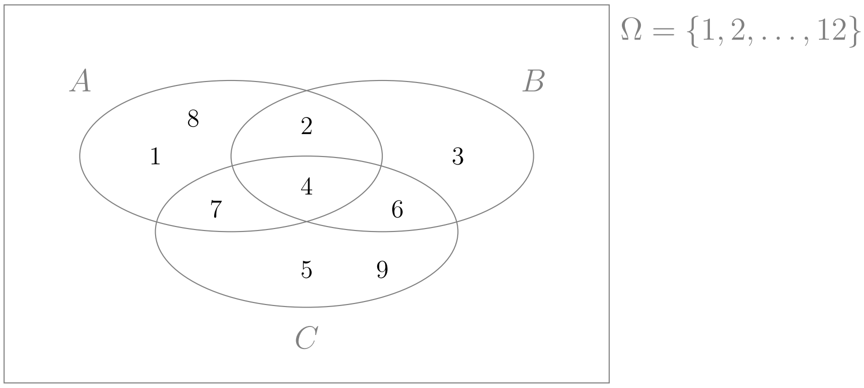
1. This question is about *repeating decimals*. Let a be the repeating decimal $0.223822382238\dots$.
   (a) Express 2238.223822382238.... as a multiple of a.
   (b) Express 2238 using a.
   (c) Express a as a fraction of two integers
   (d) Using your answer to part (d), express $0.7223822382238\dots$ as a fraction of two integers.
   (e) **Bonus:** Let $p, q, r, s, t, u, v, w \in \{0, 1, \dots, 9\}$ be digits. Express the repeating decimal
   $$0.pqrstuvwstuvwstuvw\dots$$
   as a fraction of two integers. The digits s, t, u, v, w repeat in this order after the third spot after the decimal.

2. This question is about *rational* and *irrational* numbers. Let a, b be positive rational numbers with $a < b$.
   (a) Express a and b as a ratio of positive integers.
   (b) Show that there exists a positive integer c with $0 < \frac{1}{c} < a$.
   Hint: use the ceiling function $\lceil \cdot \rceil$ to get an integer larger than or equal to a fraction.
   (c) Show that there exists $d \in \mathbf{Q}$ with $a < d < b$.
   Hint: use the arithmetic mean.
   (d) **Bonus:** Show that there exists $e \in \overline{\mathbf{Q}}$ with $a < e < b$.
   Hint: the product of a rational and an irrational number is an irrational number.

3. You are given the following sets, which contain elements of $\mathbf{N}$.



Identify the members of the following sets.

(a) $A \cup B$

(b) $C \setminus B$

(c) $C \cap A$

(d) $(A \cup C) \setminus (B \cap C)$

(e) $A \cap B \cap C$

(f) $(A \cap B \cap C) \setminus C$

(g) $\overline{A \cup B}$

(h) $\overline{A} \cup \overline{B}$