

In this worksheet we will use the following definitions.

- A **statement**, or **proposition**, is a collection of symbols that have some mathematical meaning. Examples are “ $2 + 2 = 4$ ” or “ $2 + 2 = 5$ ” or “Every prime number is the sum of three prime numbers.” Statements are denoted by letters, such as P, Q . If a statement P depends on a variable x , we write $P(x)$.
- The **value** of a statement, when it can be proved, is either *true*, denoted T , or *false*, denoted F . Some statements have no value, such as “Three number circle never plus 9.”
- The **negation** of a statement P , denoted $\neg P$, is the opposite statement, pronounced “not P ”. For example, if P is the statement “Every dog is gray”, then $\neg P$ is the statement “Not all dogs are gray.” If P is *true*, then $\neg P$ is *false*, and if P is *false*, then $\neg P$ is *true*.

1. **Warm up 1:** Complete the following table of values.

P	Q	$P \vee Q$	$P \wedge Q$	$P \implies Q$	$P \iff Q$
True	True				
True	False				
False	True				
False	False				

2. **Warm up 2:** Complete the following table of values.

P	Q	$\neg P$	$\neg P \wedge \neg Q$	$\neg P \vee Q$	$P \wedge \neg Q$	$\neg P \iff \neg Q$
True	True					
True	False					
False	True					
False	False					

3. Consider the columns in the previous two questions.

(a) Which columns in the previous two questions are the same?

(b) Which columns are exact opposites of each other?

(c) Give your responses to parts (a) and (b) as logical statements.

4. Give negations of the following statements.
- If I am older than 21 then I am older than 18.
 - You are rich or you are happy.
 - Everyone living in Rīga was born in Rīga.
 - There is a person older than 120.
 - If $|x| < |y|$, then $x^2 < y^2$.
 - If x is rational then x^2 is rational and \sqrt{x} is irrational.
5. Consider the following statements, and say whether or not they are true. Explain your reasoning, and provide a counterexample for the statements you think are false.
- There is a largest natural number.
 - For all natural numbers n , the number $3n^2 + 3n + 23$ is prime.
 - The square of an even natural number is always even.
 - The sum of two rational numbers is a rational number.
 - The sum of a rational and an irrational number is a rational number.
 - Between any two rational numbers there is another rational number.
 - Between any two rational numbers there is an irrational number.
6. Consider the following curves and find their points of intersection.
- $y = 4x + 1$ and $y = -5x + 1$
 - $y = x^2 - x - 6$ and $y = 2x^2 - 6x - 6$
 - $y = 4x^3 - 5x + 8$ and $y = 4x^3 + 5x^2 + 2$
7. Let A be a set. An **identity element** for a function $f: A^2 \rightarrow A$ is an element e of A such that $f(e, a) = a = f(a, e)$ for all a in A .
- Which of the following functions have an identity element and which do not? Those that do, what is it?
 - $(x, y) \mapsto x + y$
 - $(x, y) \mapsto x \cdot y$
 - $(x, y) \mapsto (x + y)^2$
 - Suppose that there are two elements $e_1, e_2 \in A$ that satisfy the given condition for being an identity element. Show that $e_1 = e_2$.