BBA9
BITL2

Worksheet 12

Math Lab
Fall 2020

26 November 2020

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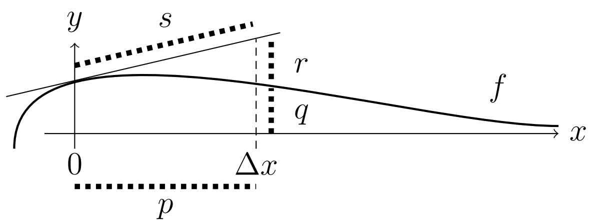
1. **Warm up:** Answer the following questions.

   (a) For which of the following limits can l'Hôpital's rule be used:

   $$\lim_{x\to 2}\frac{\sin(\pi x)}{x-2} \qquad \lim_{x\to -9}\frac{x^2-7x-18}{\ln(|x|-9)^{-1}} \qquad \lim_{x\to 0} x^x$$

   (b) The following diagram is the graph of a function f and the tangent line at $x = 0$. Which of the labeled values p, q, r, s is the *differential*?



   (c) Without graphing the functions below, how many local maxima and local minima do they have on all of **R**?

   $$x^2 \qquad x^{10} \qquad (x-4)(x-3)(x-2)(x-1)(x+1)(x+2)$$

2. Consider the function $f(x) = \frac{6}{x^2+3}$.

   (a) Find where f reaches its largest and smallest values.

   (b) Find where the slopes of tangent lines of f are steepest and flattest.

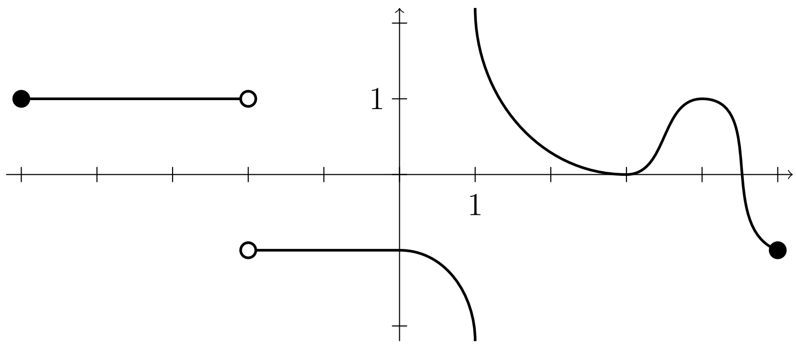
3. (a) Find the smallest minimum of $f(x) = (x-1)^2 + (x-5)^2$ and its x-value.

   (b) Find the smallest minimum of $f(x) = (x-a)^2 + (x-b)^2$ and its x-value.

   (c) Find the smallest minimum of $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$ and its x-value.

   (d) What do you think is the x-value of the smallest minimum of $f(x) = \sum_{i=1}^{n}(x-a_i)^2$?

4. Below is the graph of the derivative f' of a continuous function f on $[-5, 5]$.



(a) On what intervals is f increasing? Decreasing?

(b) On what intervals is f concave down? Concave up?

(c) What are the x-values of the local minima and maxima of f?

(d) What are the x-values of the points of inflection of f?

(e) On top of the graph above, draw a possible continuous function f that could have the graph as derivative.

5. How many maxima and minima do each of the functions have on the given interval? Find the coordinates (x, y) where these extrema occur.

(a) $y = x^2$ on $(-\infty, \infty)$

(b) $y = x(x-5)(x+5)$ on $[-6, 6]$

(c) $y = \tan(x)$ on $[-\pi/2, \pi/2]$

(d) $y = \sin(x)$ on $[0, 4\pi)$

(e) $y = e^x$ on $[-100, 100]$

(f) $y = \arctan(x)$ on $(0, \infty)$