BBA9
BITL2

Worksheet 11

Math Lab
Fall 2020

19 November 2020

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1. **Warm up:** Answer the following questions.
   (a) What is the slope of the line perpendicular to the tangent line of $y = x^2$?
   (b) How many times does the chain rule need to be applied to differentiate the following function with respect to x: $f(x) = \ln\left(\arcsin\left(\sqrt{\cos^2((e^x)^2) + 2\sin^2(e^{2x}) - 1}\right) + 1\right)$
   (c) True or False: The closer a linear approximation of f is taken to a particular value, the better estimate it will have of f of that value.

2. Evaluate the following limits using l'Hôpital's rule and other differentiation rules you know.
   (a) $\lim_{x \to 1} \frac{x^n - 1}{x - 1}$
   (b) $\lim_{z \to 0} \frac{\tan(4z)}{\tan(7z)}$
   (c) $\lim_{x \to 2^+} \frac{1}{x-2} - \frac{1}{\ln(x-1)}$
   (d) $\lim_{x \to 0^+} (e^x + x)^{\frac{1}{4x}}$
   (e) $\lim_{x \to 0^+} \frac{\sin(x) - x}{x^3}$
   (f) $\lim_{x \to 0^+} (\cos(x) - 1)^x$
   (g) $\lim_{x \to \pi/2^+} \frac{\sec(x)}{1 + \tan(x)}$
   (h) $\lim_{x \to 0^+} \frac{2\ln(e^x - 1)}{\ln(3x)}$

3. Let L_0 be the linear approximation of x^2 at 0, and L_k the linear approximation of x^2 at (k, k^2). Find the point (x, y) where L_0 intersects L_k.

4. Let $L_a(x)$ be the linear approximation to $\cos(x)$ at $x = a$.
   (a) At which value in $[0, \pi/2]$ are the differentials from $L_0(x)$ and $L_{\pi/2}(x)$ equal?
   (b) At which value in $[-\pi/2, \pi/2]$ are the differentials from $L_{\pi/2}(x)$ and $L_{-\pi/2}(x)$ equal?

5. Suppose that at price p, for $p \in (0, 10)$, the demand for a product is $f(p)$ kg, where $f(p) = 120 - 2p - p^2$.
   (a) What is the price elasticity of demand when $p = 5$?
   (b) What is the average elasticity of demand in the price interval $[5, 7]$?
   (c) Is demand for this product elastic or inelastic on the domain $(0, 10)$? Why?