BBA9
BITL2

Worksheet 9

Math Lab
Fall 2020

5 November 2020

Recall the following rules for differentiation:

- **product rule:** $\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$
- **quotient rule:** $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- **chain rule**: $\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))g'(x)$

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1. **Warm up:** Answer the following True / False questions.
   (a) If a function is differentiable at a point a, then it is continuous at a.
   (b) If a function is continuous at a point a, then it is differentiable at a.
   (c) If $f(x) = 3x^2 - 2$, then $f'(5) = f'(3) + f'(2)$.
   (d) There is no difference betwen $\frac{d}{dx}\left(5x^2 + 2xy - 3y^2\right)$ and $\frac{d}{dy}\left(5x^2 + 2xy - 3y^2\right)$.
2. Differentiate the following functions.
   (a) $4x^2 - 2x + 5/2$
   (b) $\dfrac{\sin(2x)}{3x^2 + \tan(x+1)}$
   (c) $(e^{2x-5} - 2)\left(\sqrt{6x + \sqrt{x}} - \frac{1}{x}\right)$
   (d) $\dfrac{1 + \frac{e^x}{\ln(x)}}{\frac{4x^2}{\cos(x)} - 2x}$
3. Show that the quotient rule is simply the product rule followed by the chain rule.
4. (a) Consider a circle C of radius r.
      i. What is the circumfrence of C?
      ii. What is the area of C?
      iii. What is the derivative of the area of C, with respect to r?
   (b) Consider a sphere S of radius r.
      i. What is the surface area of S?
      ii. What is the volume of S?
      iii. What is the derivative of the volume of S, with respect to r?

5. Suppose f is a differentiable function with the following value for f and f' as given below.

x	$f(x)$	$f'(x)$
0	3	-1
1	5	0
2	-2	3
3	6	1

Let $g(x) = x^2 - 3x + 2$. For each function below, calculate the derivative at the given point.

(a) $f(x) + g(x)$ at $x = 0$

(b) $\dfrac{f(x)}{g(x)}$ at $x = 1$

(c) $f(x)g(x)$ at $x = 2$

(d) $\dfrac{f(x)g(x)}{f(x) + g(x)}$ at $x = 3$

(e) $f(g(x))$ at $x = 0$

(f) $f(g(x))$ at $x = 1$

(g) $g(f(x))$ at $x = 2$

(h) $g(f(x))$ at $x = 3$

6. The hyperbolic sine and cosine are defined as $\sinh(\theta) = \frac{e^{\theta}-e^{-\theta}}{2}$ and $\cosh(\theta) = \frac{e^{\theta}+e^{-\theta}}{2}$. Regular sine and cosine may be define similarly, using the **imaginary number** i, for which $i^2 = -1$. The formulas are $\sin(\theta) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$ and $\cos(\theta) = \frac{e^{i\theta}+e^{-i\theta}}{2}$.

(a) Show that $\dfrac{d}{d\theta}\cosh(\theta) = \sinh(\theta)$ and $\dfrac{d}{d\theta}\sinh(\theta) = \cosh(\theta)$.

(b) Using the sum and difference formula for $\sin(a \pm b)$ and $\cos(a \pm b)$, express $\sin(x + iy)$ and $\cos(x + iy)$ with the exponential function.