

29 October 2020

1. **Warm up:** What are the derivatives of the following functions?

(a)  $x^2$

(c)  $2^x$

(e)  $x^{-2}$

(b)  $x + 2$

(d)  $(x^{1/2})^{1/2}$

(f)  $e^{2x}/2^{ex}$

2. Recall that  $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$ . Using this, evaluate the following limits.

(a)  $\lim_{t \rightarrow 0} (1 + t)^{1/t}$

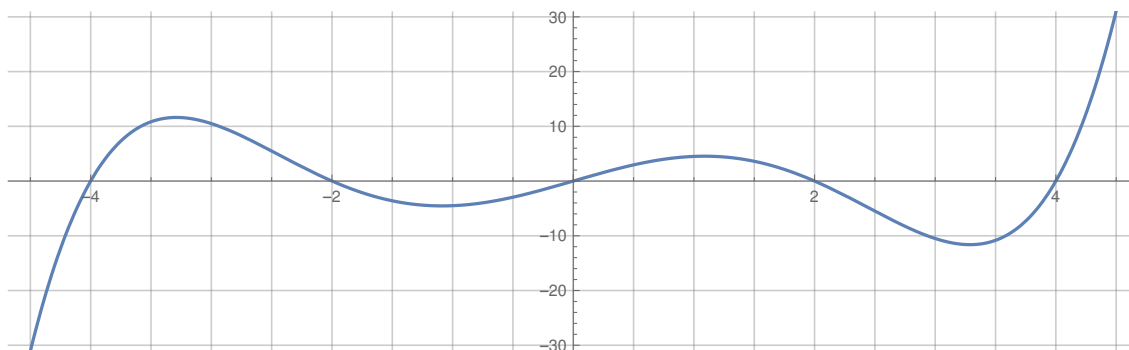
(c)  $\lim_{x \rightarrow \infty} \left( \frac{2x - 3}{2x + 1} \right)^{\frac{x+1}{3}}$

(b)  $\lim_{n \rightarrow \infty} \left( 1 + \frac{6}{2 - n} \right)^{n+1}$

(d)  $\lim_{n \rightarrow -\infty} \left( \frac{n}{n-1} \right)^{2n}$

3. Use the limit definition of the derivative to find  $f'(2)$  for  $f(x) = 2x - 4x^{-1}$ .

4. Below is the graph of  $f(x) = \frac{x}{10}(x+2)(x-2)(x+4)(x-4)$ .



(a) Compute the derivative  $f'(x)$ .

(b) At what  $x$ -values will the tangent line to  $f$  be horizontal?

(c) What will be the  $y$ -values at the given  $x$ -values from part (b)?

5. Draw the graph of a continuous function  $f$  that satisfies the following conditions:

- $f'(-1) = f'(1) = 0$
- $f'(-2) = f'(2) = 1$
- $f'(x) < 0$  when  $x \leq -3$  and  $f'(x) > 0$  when  $x \geq 3$

6. Let  $f(x) = 2x^2 + 5$  and  $g(x) = 4e^x - 1$ .

(a) What is the function  $f(g(x))$ ? What is  $g(f(x))$ ?

(b) Compute the derivative  $f'(x)$  and  $g'(x)$ .

(c) What is  $f'(g(x))$ ? What is  $g'(f(x))$ ?