BBA9 **BITL2**

Worksheet 6

Math Lab **Fall 2020**

15 October 2020

Recall the **instantaneous rate of change** of a function f at a point a in its domain is

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

This is a **limit**. There are also **one-sided** limits, written $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$, where the value $f(a)$ is approached from the left (a^-) or from the right (a^+). Recall also:

- the function f has a **horizontal asymptote** at $y=a$ if $\lim_{x\to\infty} f(x)=a$ or $\lim_{x\to-\infty} f(x)=a$
- the function f has a **vertical asymptote** at $x=a$ if $\lim_{x\to a^\pm} f(x)=\pm\infty$.

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1. **Warm up:** Answer the following True / False questions.
   (a) If $\lim_{x\to 0} f(x)=1$, then $\lim_{x\to 0}(f(x)-1)=0$.
   (b) If $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} 2f(x)\geqslant \lim_{x\to 0} f(x)$.
   (c) For any two functions f and g, $\lim_{x\to 5} f(x)+\lim_{x\to 5} g(x)=\lim_{x\to 5}(f(x)+g(x))$.

2. Evaluate the following limits.
   (a) $\lim_{x\to 9}\dfrac{2x^2-3}{9x}$
   (b) $\lim_{y\to 4}\dfrac{y^2-y-12}{\sqrt{y}-2}$
   (c) $\lim_{z\to 3}\dfrac{3-z}{z-3}$
   (d) $\lim_{x\to 0} x\sin(x)$

3. For each of the functions f below, identify their vertical asymptotes $x=a$, and evaluate $\lim_{x\to a^\pm} f(x)$.
   (a) $\dfrac{x^2+3x-1}{x+2}$
   (b) $\dfrac{x^2-10x+16}{x-2}$
   (c) $\dfrac{3x}{(5+6x+x^2)^2}$
   (d) $\tan\left(\dfrac{\pi x}{2}\right)$

4. The **floor function** $f(x)=\lfloor x\rfloor$ gives the largest integer less than or equal to x.
   (a) What is the domain of f?
   (b) Where does $\lim_{x\to a} f(x)$ exist?
   (c) Where does $\lim_{x\to a^+} f(x)$ exist?
   (d) Where does $\lim_{x\to a^-} f(x)$ exist?

5. Let $a,b,c\in\mathbf{R}$ be distinct.
   (a) Create a function $f(x)$ with $\lim_{x\to\infty} f(x)=a$.
   (b) Create a function $g(x)$ with $\lim_{x\to\infty} g(x)=a$ and $\lim_{x\to-\infty} g(x)=b$.
   (c) Create a function $h(x)$ with $\lim_{x\to\infty} h(x)=a$ and $\lim_{x\to-\infty} h(x)=b$ and $\lim_{x\to c} h(x)=\infty$.