BBA9
BITL2

Worksheet 4

Math Lab
Fall 2020

1 October 2020

Recall the following definitions. for a function $f\colon \mathbf{R} \to \mathbf{R}$.

- The function f is **odd** if $f(x) = -f(-x)$ for all $x \in \mathbf{R}$.
- The function f is **even** if $f(x) = f(-x)$ for all $x \in \mathbf{R}$.

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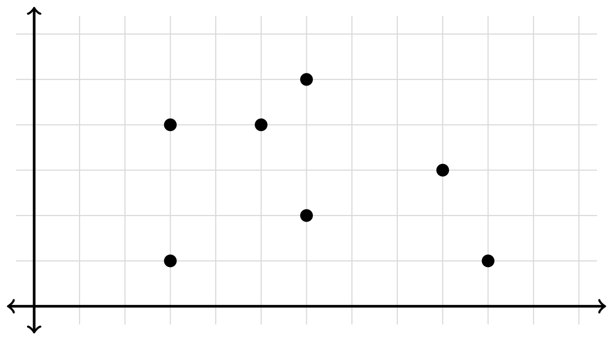
1. **Warm up:** Consider these 7 points in the plane. How many different linear functions can be drawn that pass through at least two of them?



2. Consider the functions $f(x) = x^2$, $g(x) = 2x - 1$ and $h(x) = \frac{x+2}{2}$. Draw graphs of the following functions:

$f(x)$	$h(g(x))$
$f(g(x))$	$g(h(x))$
$h(f(g(x)))$	$f(g(h(x)))$

3. Let $f(x) = ax + b$ and $g(x) = cx + d$ be two linear functions, for $a, b, c, d \in \mathbf{R}$, with $a \neq c$.
   (a) For some point $x \in \mathbf{R}$, what is the distance in the plane between $f(x)$ and $g(x)$?
   (b) For what value x is this distance as small as possible?
   (c) Let C be a circle with center on the y-axis. What is its equation if it intersects f and g at their y-intercepts?
   (d) Let C' be a circle with center on the line $x = a$. What is its equation if it intersects f and g at $f(a)$ and $g(a)$, respectively?

4. Consider the function $f(x) = \begin{cases} x^2 - 1 & \text{if } x \leqslant -1, \\ ax + b & \text{if } -1 < x < 1, \\ |x| & \text{if } x \geqslant 1. \end{cases}$

   Find a pair of real numbers a, b that make the function f have no jumps.

5. Prove by induction that $\dfrac{x^n - 1}{x - 1} = 1 + x + x^2 + \cdots + x^{n-1}$ for all $n \in \mathbf{N}$.