

10 September 2020

In this worksheet we will use the following definitions.

- A **statement**, or **proposition**, is a collection of symbols that have some mathematical meaning. Examples are “ $2 + 2 = 4$ ” or “ $2 + 2 = 5$ ” or “Every prime number is the sum of three prime numbers.” Statements are denoted by letters, such as P, Q . If a statement P depends on a variable x , we write $P(x)$.
- The **value** of a statement, when it can be proved, is either *true*, denoted T , or *false*, denoted F . Some statements have no value, such as “Three number circle never plus 9.”
- The **negation** of a statement P , denoted $\neg P$, is the opposite statement, pronounced “not P ”. For example, if P is the statement “Every dog is gray”, then $\neg P$ is the statement “Not all dogs are gray.” If P is *true*, then $\neg P$ is *false*, and if P is *false*, then $\neg P$ is *true*.

1. **Warm up 1:** Complete the following table of values.

P	Q	$P \vee Q$	$P \wedge Q$	$P \implies Q$	$P \iff Q$
True	True				
True	False				
False	True				
False	False				

2. **Warm up 2:** Complete the following table of values.

P	Q	$\neg P$	$\neg P \wedge \neg Q$	$\neg P \vee \neg Q$	$P \wedge \neg Q$	$\neg P \iff \neg Q$
True	True					
True	False					
False	True					
False	False					

3. Consider the columns in the previous two questions.

- Which columns in the previous two questions are the same?
- Which columns are exact opposites of each other?
- Give your responses to parts (a) and (b) as logical statements.

4. Give negations of the following statements.

- (a) If I am older than 21 then I am older than 18.
- (b) You are rich or you are happy.
- (c) Everyone living in Rīga was born in Rīga.
- (d) There is a person older than 120.
- (e) If $|x| < |y|$, then $x^2 < y^2$.
- (f) If x is rational then x^2 is rational and \sqrt{x} is irrational.

5. Consider the following statements, and say whether or not they are true. Explain your reasoning, and provide a counterexample for the statements you think are false.

- (a) There is a largest natural number.
- (b) For all natural numbers n , the number $3n^2 + 3n + 23$ is prime.
- (c) The square of an even natural number is always even.
- (d) The sum of two rational numbers is a rational number.
- (e) The sum of a rational and an irrational number is a rational number.
- (f) Between any two rational numbers there is another rational number.
- (g) Between any two rational numbers there is an irrational number.

3. Consider the following curves and find their points of intersection.

- (a) $y = 4x + 1$ and $y = -5x + 1$
- (b) $y = x^2 - x - 6$ and $y = 2x^2 - 6x - 6$
- (c) $y = 4x^3 - 5x + 8$ and $y = 4x^3 + 5x^2 + 2$

4. Let A be a set. An **identity element** for a function $f: A^2 \rightarrow A$ is an element e of A such that $f(e, a) = a = f(a, e)$ for all a in A .

(a) Which of the following functions have an identity element and which do not? Those that do, what is it?

- i. $(x, y) \mapsto x + y$
- ii. $(x, y) \mapsto x \cdot y$
- iii. $(x, y) \mapsto (x + y)^2$

(b) Suppose that there are two elements $e_1, e_2 \in A$ that satisfy the given condition for being an identity element. Show that $e_1 = e_2$.