ESP Math 294 | # Worksheet 10 | **Fall 2018**

Week of 29 October 2018

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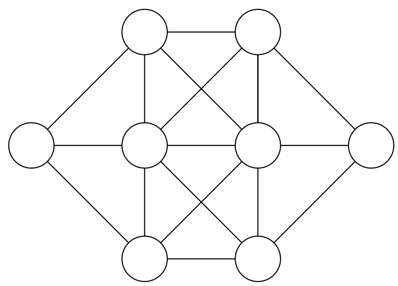
1. Draw all connected graphs with 4 vertices that are not isomorphic to each other.

2. Label the vertices of the graph below by a, b, c, d, e, f, g, h so that no two consecutive letters are connected by an edge.



3. Let G be a connected planar graph. Every planar graph decomposes the plane into a number of pieces, called **faces**. Prove that

$$(\text{number of vertices of } G) - (\text{number of edges of } G) + (\text{number of faces of } G) = 2.$$

Hint: Use induction on the number of faces.

4. Recall that a set A is **countable** if it is finite or if there exists a bijection $A \to \mathbf{N}$.

   (a) Let A and B be countable. Prove that $A \cup B$ is countable.

   (b) Let $A_1, A_2, A_3, \ldots$ be countable sets. Prove that $\bigcup_{i=1}^{\infty} A_i$ is countable.

5. Recall a **polynomial** is a function $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$, where $n \in \mathbf{Z}_{\geqslant 0}$ is the **degree** of p and the $a_i \in \mathbf{R}$ are the **coefficients** of p.

   (a) Prove that the set of all polynomials with integer coefficients is countable.

   (b) A real number $\alpha \in \mathbf{R}$ is **algebraic** is there exists a polynomial $p(x)$ with integer coefficients such that $p(\alpha) = 0$. Prove the set of all algebraic numbers is countable.