Worksheet 9

Week of 22 October 2018

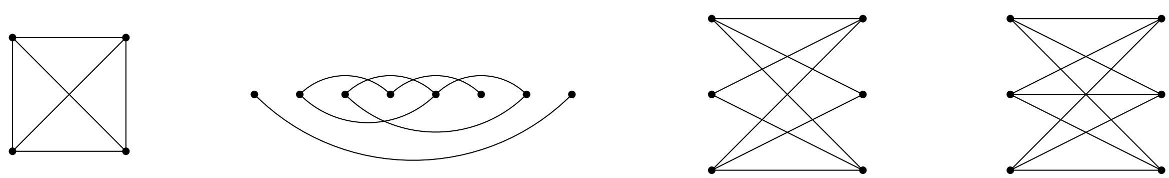
A **graph** G is a pair of sets (V, E), such that every element of E is a pair $\{v_1, v_2\}$, for $v_1, v_2 \in V$. The elements of V are called **vertices** and the elements of E are called **edges**.

- Graphs $G = (V, E)$ and $H = (W, F)$ are **isomorphic** if there is a bijection $f\colon V \to W$ so that $\{v_1, v_2\} \in E$ if and only if $\{f(v_1), f(v_2)\} \in F$.
- A **subgraph** of a graph $G = (V, E)$ is a graph $H = (W, F)$ with $W \subseteq V$ and $F \subseteq E$. The subgraph H of G is **spanning** if $W = V$.
- A **path** is a sequence of edges $\{e_1, \dots, e_n\}$ so that e_i and e_{i+1} share exactly one element in common.

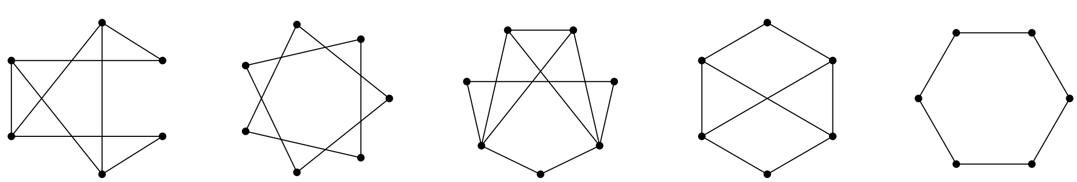
A graph $G = (V, E)$ is
connected if for every pair $v_1, v_2 \in V$ there is a path from v_1 to v_2,
k**-regular** if every vertex $v \in V$ occurs in exactly k elements of E,
planar if it can be drawn with no edge crossings.

---

1. Which of the following graphs are planar? Which are not?



2. (a) For each graph below, find a connected spanning subgraph. Make it as small as possible.
   (b) Which of the graphs below are isomorphic?
   (c) Which of the graphs below contain others as subgraphs?



3. Give an example of a connected 5-regular graph with 6 vertices.
4. A **Hamilton cycle** of a graph is a 2-regular connected spanning subgraph. By considering all possible choices, show that the Petersen graph does not have a Hamilton cycle.
5. A **matching** of a graph $G = (V, E)$ is a subset $M \subseteq E$ so that no vertex of V appears twice in M. A matching is **perfect** if all vertices of V appear in M.
   (a) Which of the graphs on this page have perfect matchings? Which do not?
   (b) Draw an example of a 3-regular graph that does not have a perfect matching.