ESP Math 294 | # Worksheet 7 | **Fall 2018**

Week of 8 October 2018

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1. A **binary relation** on a set A is a function $r \colon A \times A \to \{T, F\}$. Let $A = \{1, 2, 3\}$.
   (a) How many binary relations are there on A?
   (b) How many reflexive binary relations are there on A?
   (c) How many symmetric binary relations are there on A?
   (d) How many anti-symmetric binary relations are there on A?
   (e) **Bonus:** How many equivalence relations are there on A?
   (f) **Bonus:** How many transitive binary relations are there on A?
2. Let $n \in \mathbf{N}$. Prove that $a \equiv b \pmod{n}$ is an equivalence relation on $a, b \in \mathbf{Z}$.
3. (a) Prove that 123456789 is not equal to $n!$ for any $n \in \mathbf{N}$.
   (b) Prove that $\log_{10}(123456789)$ is not a rational number.
4. Find all pairs of integers (x, y) such that $xy = 12 + 2x$.
5. Find the remainder when 64^{122} is divided by 31. You may assume **Fermat's little theorem**, which states that for $n \in \mathbf{Z}$ and p prime, $n^p \equiv n \pmod{p}$. If n and p are coprime, then we also have $n^{p-1} \equiv 1 \pmod{p}$.
6. Prove that if $x \equiv y \pmod{m}$ and $x \equiv y \pmod{n}$, then $x \equiv y \pmod{mn}$, for $\gcd(m, n) = 1$.