ESP Math 294 **Fall 2018**

Worksheet 6

Week of 1 October 2018

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1. Let $f\colon A \to B$ and $g\colon B \to C$ be two functions of sets. Their **composition** is the function $g \circ f\colon A \to C$, defined by $(g \circ f)(a) := g(f(a))$.
   (a) Suppose that f and g are injective. Prove that $g \circ f$ is injective.
   (b) Suppose that f and g are surjective. Prove that $g \circ f$ is surjective.
   (c) Give an example of A, B, C, f, g so that f is not surjective, but g and $f \circ g$ are surjective.

2. Let $\epsilon > 0$ be a real number. Prove that there exists $n \in \mathbf{N}$ such that $0 < \frac{1}{n} < \epsilon$.

3. Use induction to show that $\dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + \cdots + \dfrac{n}{2^n} = 2 - \left(\dfrac{n+2}{2^n}\right)$ for all positive integers n.

4. Prove that every integer $\geqslant 12$ can be written in the form $3x + 7y$, where x and y are non-negative integers.

5. A sequence $\{a_n\}$ of real numbers is **convergent** if there exists $L \in \mathbf{R}$ such that

$$\epsilon > 0 \quad \Longrightarrow \quad \exists\ N \in \mathbf{N} \text{ such that } |a_n - L| < \epsilon\ \forall\ n \geqslant N.$$

   For the given convergent sequences and values of ϵ, find L and N satisfying this statement.
   (a) $a_n = \frac{\sin(n)}{n}$ and $\epsilon = 1/50$
   (b) $a_n = \frac{n}{n+1}$ and $\epsilon = 0.0001$