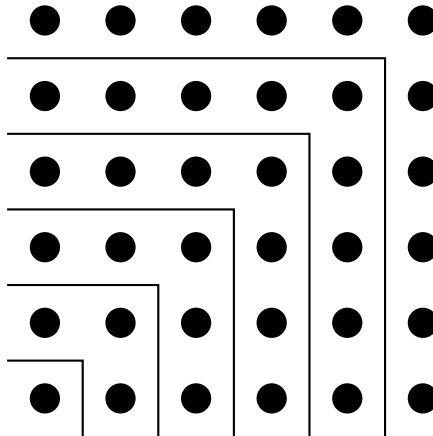


1. Consider the picture below.



(a) Use this to compute $1 + 3 + 5 + 7 + 9$. What does $1 + 3 + \dots + 97 + 99$ equal?

(b) Generalize this pattern to dots in a triangular shape.

2. Suppose we tie a rope tightly around the Earth's equator, then add 10 miles to the rope's length. Then all around the Earth the rope is raised up uniformly as high as possible to make it tight again.

(a) How high off the Earth's surface is the rope? Assume Earth has a radius of 4000 miles.

(b) What if the same is done on the Moon? The Moon has a radius of about 1000 miles.

3. Describe the following terms, as precisely as you can:

(a) natural number

(c) rational number

(b) integer

(d) irrational number

4. Consider the five statements below.

- (P_1) I like pineapple on my pizza.
- (P_2) All odd-numbered statements are false.
- (P_3) All even-numbered statements are true.
- (P_4) At least one of P_2 or P_3 is true.
- (P_5) If P_1 is false then P_2 is true.

Some of these statements refer to other statements on the list. Notice that if P_3 is true then all even-numbered statements must be true, and so P_2 must be true. The truth of P_2 implies all odd-numbered statements are false, and so P_3 is false. So if P_3 is true then it must also be false. This contradiction means P_3 must not be true.

Assign the truth values *True* and *False* to each of the above five statements so that there are no contradictions, or say why it is not possible.

5. Prove or disprove the following statements:

- (a) For all natural numbers n , the number $3n^2 + 3n + 23$ is prime.
- (b) There is a largest natural number.
- (c) The sum of two rational numbers is a rational number.
- (d) The sum of a rational and an irrational number is a rational number.
- (e) Between any two rational numbers there is another rational number.
- (f) Between any two rational numbers there is an irrational number.