ESP Math 182 **Spring 2019**

Mock final

30 April 2019

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1. *Integral methods:* Evaluate the following integrals. Show all your work.

   (a) $\displaystyle\int \frac{x^2 e^{\sqrt{x^3-3}}}{\sqrt{x^3-3}}\,dx$

   (b) $\displaystyle\int x^2 \sin(2x-5)\,dx$

   (c) $\displaystyle\int (\csc(3x)+\cot(3x))^2\,dx$

   (d) $\displaystyle\int_5^7 \frac{x+1}{9x^2+4}\,dx$

   (e) $\displaystyle\int_e^3 \frac{x^2+x-20}{x^3-4x^2+4x}\,dx$

   (f) $\displaystyle\int_1^{2\pi} e^x \cos(x)\,dx$

2. *Area between curves:* Find the integral that represents the area above the curve $y = (x-3)^2 - 12$ and below both of the curves $y = (x-2)^3 + 5$ and $y = 7 - x$. Do not evaluate the integral. *Hint: The cubic and linear curves intersect at $x = 2$.*

3. *Volumes of revolution:* Calculate the following volumes using the disk method.

   (a) The area bounded by $y = \ln(x)$, $y = 4 - \ln(x)$, $x = 2$, and $x = 4$ revolved around the x-axis.

   (b) The area in the second quadrant bounded by $x = -y^2$ and $y = x^2$ revolved around the axis $y = -3$.

   (c) The volume of revolution of $y = x(x-1)(x-2)$ revolved around the x-axis between $x = 0$ and $x = 3$.

4. *Sequences:* For each of the following sequences, determine if it converges or diverges. If it converges find the limit.

   (a) $x_n = \dfrac{n}{n+1}$ (b) $x_n = \dfrac{n\cos(n\pi)}{2n+1}$ (c) $x_n = \dfrac{\sin(n)}{n}$

5. *Series - convergence / divergence tests:* Determine if the following series converge or diverge. Indicate which tests you have used.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{\sqrt{n^2+1}}$ (b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n}{12} - \frac{n+1}{6}\right)$

6. *Series - sum of series:* Find the value of the following convergent series. Indicate what type of series they are.

   (a) $\displaystyle\sum_{n=0}^{\infty} 2^{2n} 4^{3n+1} e^{8-8n}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{n^2+4n+3}$

7. *Series - intervals of convergence:* Find the intervals of convergence of the following series. Indicate which tests you have used.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(x-2)^n}{(n\ln(n))^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{15^n n}$

8. *Power series:*

   (a) Find the first four terms of the Maclaurin series of $f(x) = \int_0^x 3t^3 - \frac{5}{2}t^2 + 2\ dt$.

   (b) What are the Maclaurin series of the following common functions?

   i. e^x ii. $\frac{1}{1+x}$ iii. $\cos(x)$

   (c) Find the first three terms of the Taylor series of $f(x) = 2e^{2x}\sin(2x)$ at $x = a$.

9. *Parametric equations:*

   (a) Describe the linear system

   $$\begin{aligned} 4x + 5y - 2z &= 7, \\ x - y + 10z &= 1 \end{aligned}$$

   as a parametric equation in the variable t.

   (b) For the parametric curve $(x, y) = (5t - 2, 8 - 3t)$, find $\frac{dy}{dx}$ and the values of t for which the graph is in the first quadrant.

10. *Matrices:* Find the determinant, eigenvalues, and eigenvectors of the matrix $\left[\begin{smallmatrix} 1 & 1 \\ -1 & 2 \end{smallmatrix}\right]$.