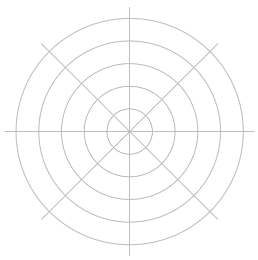
Worksheet 20

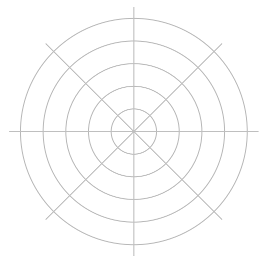
16 April 2019

1. **Warm up 1:** Draw the regions described by the inequalities in polar coordinates below.

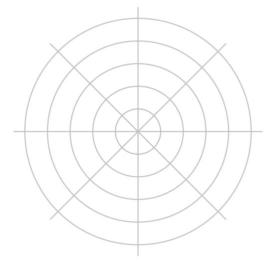
   (a) $r \leqslant 3$, $\pi \leqslant \theta < 2\pi$

   (b) $r < 3$, $\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{2}$

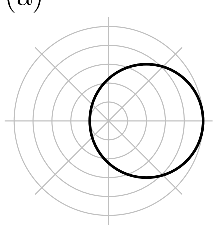
   

   (c) $3 \leqslant r \leqslant 5$, $\frac{3\pi}{4} \leqslant \theta \leqslant \pi$

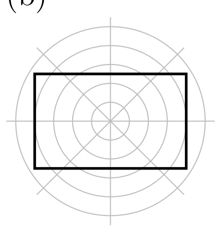
   

2. Determine which of the shapes below could be described as polar functions $r = f(\theta)$ and which can not. (*Bonus: Find the polar functions that have them as graphs.*)

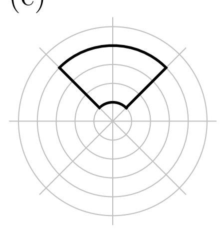
   (a)

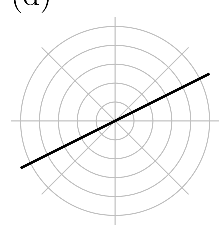
   (b)

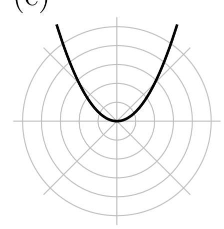
   (c)

   (d)

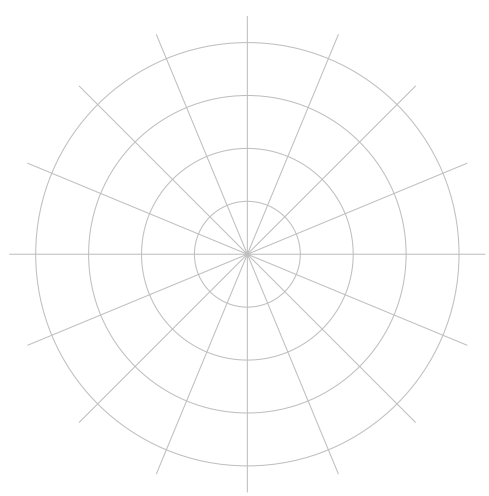
   

   (e)

3. Consider the polar function $r = \cos(8\theta) + 3$.

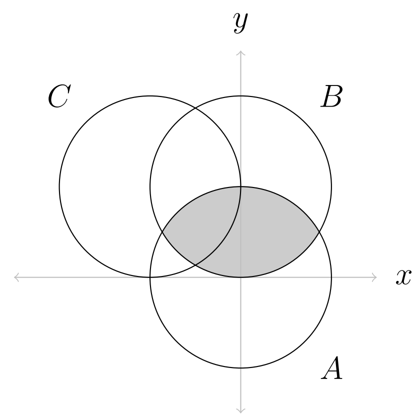
   (a) On the polar plot below, draw all the points $(f(\theta), \theta)$, for $\theta = 0, \pi/8, ..., 2\pi$. That is, draw the values of the function at intervals of $\pi/8$.

   (b) Connect the dots so that you see what the function looks like.

   (c) What can the coefficient 8 in $\cos(8\pi)$ be changed to so that $f(0) \neq f(2\pi)$?

4. (a) Describe the three unit circles below as polar equations $r = f(\theta)$.

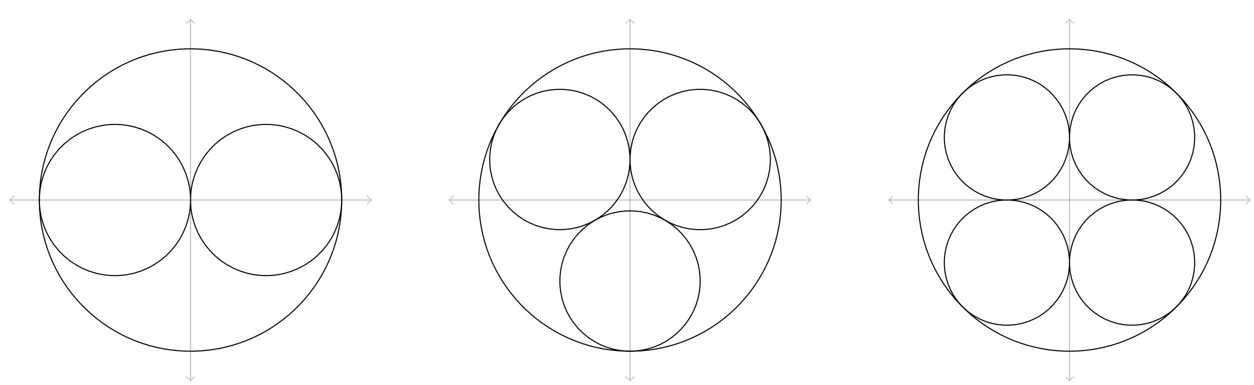


(b) Find the area inside both A and B (the shaded area) using a polar integral. Be careful with the bounds, make sure you know which parts of the curves you are integrating.

(c) Find the area inside A but not inside B in the first quadrant.

(d) Describe the area inside both A and C by an integral, but do not solve the integral.

5. Find the areas of the two, three, and four inscribed circles (all inside unit circles) below.



Which collection of inscribed circles covers the most area?