ESP Math 182

Worksheet 17

Spring 2019

14 March 2019

---

1. **Warm up:** Let $f(x) = \begin{cases} e^{-1/x^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$

   (a) Find $f'(0)$, $f''(0)$, and $f'''(0)$.

   (b) For what values of x is $f(x) = 0$?

   (c) Make a guess as to what is the Taylor series of f at 0.

2. Calculate the partial sums S_3, S_4, and S_5, then find the sum of the telescoping infinite series

$$\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right).$$

3. Use partial fractions to rewrite the general term for this series. Then compute its value.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$$

4. Use the comparison test to determine whether the series converge or diverge.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 + \sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 2^n}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{2^n \ln(n)}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^2}$

5. Use the alternating series test to determine whether the series converge or diverge.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-11}\sqrt{n}}{n+11}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{n^{1/4}}$

6. Use any tests you know to determine whether the series converge or diverge.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2 + 1}{\sqrt{n^7 + n - 1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^n}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$