

12 March 2019

1. **Warm up:** How many times can the following functions be repeatedly differentiated, at  $x = 0$ ?

(a)  $f(x) = x$

(d)  $\alpha(x) = x + |x| + \sin(x)$

(b)  $g(x) = |x|$

(e)  $\beta(x) = \begin{cases} 0 & x \leq 0, \\ x^3 & x > 0. \end{cases}$

(c)  $h(x) = \sin(x)$

2. Are the following expressions power series in  $x$ ? Why or why not?

(a)  $\sum_{n=1}^{55} \frac{1}{n}$

(e)  $\sum_{n=0}^{\infty} x^n n^x$

(b)  $\sum_{n=1}^{\infty} \frac{1}{n}$

(f)  $\{1, 2, 4, 8, 16, \dots\}$

(c)  $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(g)  $\{x, x^2, x^4, x^8, x^{16}, \dots\}$

(d)  $\sum_{n=1}^{\infty} \sum_{k=1}^n \frac{x^n}{n^k}$

(h)  $1 + 2 + 4 + 8 + 16 + \dots$

(i)  $1 + x^2 + x^4 + x^8 + x^{16} + \dots$

(j)  $\pi$

(k)  $e^x$

3. Consider the function  $f(x) = 4x^3 - 2x^2 + 3x - 1$ .

(a) Find the 2nd order Taylor polynomial for  $f$  at  $x = 0$ .

(b) Find the interval on which the Taylor approximation is no more than 0.5 away from the function.

4. For each of the following collections of points, find two different functions that go through those points.

(a)  $(0, 0), (1, 0)$

(c)  $(0, 0), (\pi, 0), (2\pi, 0)$

(b)  $(0, 1), (2, 1)$

(d)  $(0, 0), (1, 1), (2, 2), (3, 3)$

5. Consider the series  $\sum_{n=1}^{\infty} \frac{n^2}{6^n}$ .

(a) Is this a geometric series? Why or why not?

(b) Find a geometric series that bounds this series above.

(c) Can you conclude that the original series converges?