

5 March 2019

1. **Warm up:** Answer the following True / False questions.

(a) $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is a geometric series.

(b) $\int_1^{100} \frac{1}{x-1}$ is an improper integral.

(c) The sequence $1, -1, 1, -1, 1, -1, \dots$ diverges.

2. This question is about the *integral test* for series, which you may not have seen in class. The test states that for a non-increasing function f ,

$$\sum_{n=N}^{\infty} f(n) \text{ converges} \iff \int_N^{\infty} f(x) dx \text{ is finite.}$$

Use the integral test for the questions below.

(a) Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

(b) Show that $\sum_{n=1000}^{\infty} \frac{1}{n}$ diverges.

(c) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$.

(d) Calculate $\sum_{n=1}^{100} \frac{1}{e^n}$ and $\int_1^{100} \frac{1}{e^x} dx$.

3. Using any convergence / divergence test you have learned to determine if the following series converge or diverge. Say which test(s) you are using.

$$(a) \sum_{n=1}^{\infty} \frac{n+1}{n}$$

$$(g) \sum_{n=1}^{\infty} \frac{1}{2+3^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{n+1}{n^3}$$

$$(h) \sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$$

$$(c) \sum_{n=1}^{\infty} \frac{n^2}{n^3+4}$$

$$(i) \sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$

$$(d) \sum_{n=1}^{\infty} \frac{2}{4n^2-1}$$

$$(j) \sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$$

$$(e) \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$$

$$(k) \sum_{n=1}^{\infty} \sin(1/n^2)$$

$$(f) \sum_{n=1}^{\infty} \frac{1}{2+\sqrt{3n}}$$

$$(l) \sum_{n=1}^{\infty} \frac{n!}{4^n n^3}$$