ESP Math 182

Worksheet 13

Spring 2019

28 February 2019

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1. **Warm up:** Answer the following true / false questions.

   (a) The sequence $\{a_n\}_{n=1}^{\infty}$ for $a_n = \frac{1}{n}$ converges.

   (b) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

   (c) If a series $\displaystyle\sum_{n=0}^{\infty} a_n$ converges and $a_n \to c$ as $n \to \infty$, then $c = 0$.

   (d) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges to 0, then $\displaystyle\sum_{n=0}^{\infty} a_n$ converges.

2. Determine if the following infinite series converge. If so, find the sum.

   (a) $\displaystyle\frac{1}{10} + \frac{3}{20} + \frac{9}{40} + \frac{27}{80} + \frac{81}{160} + \cdots$

   (b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n 2^{2-3n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n e^{3-n} 2^{n+1} - \left(\frac{2}{3}\right)^{2n}$

   (d) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{3}\right)^{2n} + \frac{3 \cdot 8^n}{81^{n/2}}$

   (e) $\displaystyle\frac{3}{4} + \frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{108} + \cdots$

3. Notice that $0.9 = \frac{9}{10}$, $0.99 = \frac{9}{10} + \frac{9}{100}$ and so on.

   (a) Use this pattern to define a sequence $\{a_n\}$ such that $\displaystyle\sum_{n=1}^{\infty} a_n = 0.99999\ldots.$

   (b) Use this pattern to define a sequence $\{a_n\}$ such that $\displaystyle\sum_{n=1}^{\infty} a_n = 0.1234123412\ldots.$

4. Use geometric series to show that:

   (a) $0.99999\ldots.. = 1$ (b) $0.5555555\ldots.. = 5/9$ (c) $1.36363636\ldots. = 15/11$

5. Recall that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

(a) Reindex this series so that the index starts at $n = 0$. That is, keep the series the same, but change the $\frac{1}{n}$ to something else.

(b) Use part (a) to show that $\sum_{n=1}^{\infty} \frac{1}{n+1}$ diverges.

(c) Use these ideas to show that, for any positive integer k, the series $\sum_{n=1}^{\infty} \frac{1}{n+k}$ diverges.

6. Determine if the following statements are true or false. If true, provide some justification. If false, provide a counterexample.

(a) $\sum_{n=0}^{k}(a_n + b_n) = \sum_{n=0}^{k} a_n + \sum_{n=0}^{k} b_n$ for $k < \infty$

(b) $\sum_{n=0}^{\infty}(a_n + b_n) = \sum_{n=0}^{\infty} a_n + \sum_{n=0}^{\infty} b_n$

(c) $\sum_{n=0}^{\infty} a_n b_n = \left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right)$