ESP Math 182

Worksheet 11

Spring 2019

21 February 2019

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1. This exercise is an introduction to the *Fourier transform* of a function. For a function f that satisfies $f(0) = f(1)$, define its Fourier transform as the function

$$\hat{f}(t) = \int_0^1 f(x)e^{-2\pi i t x}\, dx,$$

where $i = \sqrt{-1}$ is the *imaginary number*.

(a) Show that $\widehat{f'}(x) = 2\pi i t \hat{f}(x)$. Hint: use integration by parts and that $f(0) = f(1)$.

(b) Let g be some function, and let f be described by the *differential equation*

$$\frac{f''(x)}{4\pi^2} + 4f(x) = g(x).$$

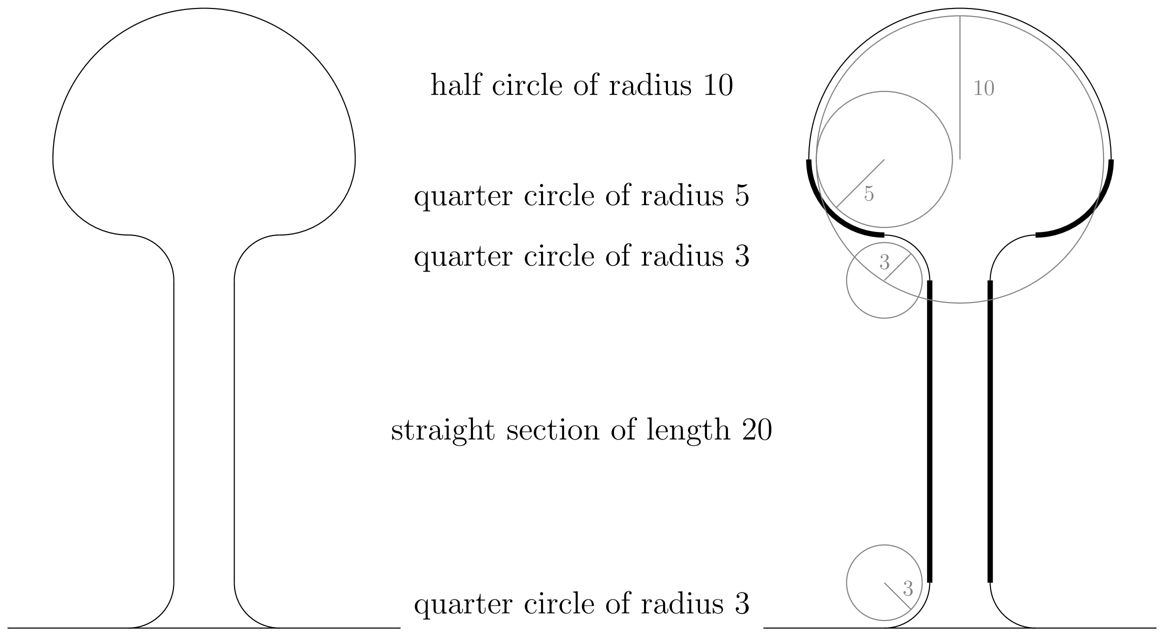
Apply the Fourier transform (take the integral of both sides) to solve for $\hat{f}$.

(c) Let f be a function defined on $[0, 1]$ as

$$f(x) = \begin{cases} x & 0 \leqslant x \leqslant 1/2, \\ -x + 1 & 1/2 \leqslant x \leqslant 1. \end{cases}$$

Calculate the Fourier transform $\hat{f}$ and find $\hat{f}(1)$.

2. Consider a cutaway of a symmetric water tower below, with units in feet.



Given the geometric shapes of the walls, **write down** (**do not evaluate!**) the integrals and expressions that give the total volume of water that could be contained within the 3-dimensional structure.