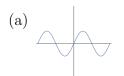
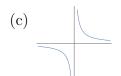
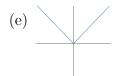
Worksheet 10

19 February 2019

1. Warm Up: Assign to each graph below its function.

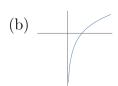


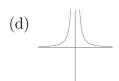


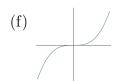


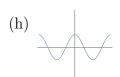


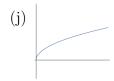












$$x^3 \qquad \cos(x)$$

 $\arctan(x)$

 $|x| \qquad \sqrt{x}$

 $e^x \qquad \ln(x)$

1/x

 $\sin(x)$

 $1/x^{2}$

- 2. Consider the integral $\int_1^\infty \frac{1}{x^p} dx$.
 - (a) Evaluate the integral for p > 1.
 - (b) Evaluate the integral for p = 1.
 - (c) Evaluate the integral for p < 1.
 - (d) What can you conclude about the finiteness of the integral, with respect to p?

3. Use integration by parts to show that $\int_1^\infty \frac{\sin(t)}{\sqrt{t}} dt$ is finite.

Hint: For IBP, integrate $\sin(t)$ and differentiate $\frac{1}{\sqrt{t}}$. Then use $-1 \le \cos(x) \le 1$. Keep in mind the result of question 2. above.

4. Suppose f is continuous. Show that if $\int_1^\infty f(x) dx$ is infinite, then $\int_2^\infty f(x) dx$ is infinite.

5. Evaluate each of the following definite integrals.

(a)
$$\int_{0}^{\pi/2} \sin^2(3x) \ dx$$

(d)
$$\int_0^{\pi^2} \sin(\sqrt{x}) \ dx$$

(b)
$$\int_{1}^{2} x \sqrt{x-1} \ dx$$

(e)
$$\int_{-2}^{2} (t+3)\sqrt{4-t^2} dt$$

(c)
$$\int_0^{\ln(3)} x^2 e^{-x} dx$$

$$(f) \int_0^3 \frac{dx}{\sqrt{9+x^2}}$$