ESP Math 182 | **Spring 2019**

Worksheet 9

14 February 2019

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1. **Warm Up:** A polynomial is a function $f(x) = a_0 + a_1 x + \cdots + a_n x^n$ where $n \in \mathbf{Z}_{\geqslant 0}$ and $a_i \in \mathbf{R}$. Using this definition, decide which of the following functions are polynomials.

   (a) $f(x) = 0$

   (b) $g(x) = 3x + \frac{5}{2}$

   (c) $h(y) = 55y^5 + \frac{\pi^3 y^4}{e^2} + 3y^3 + 22y^2 - 2015.2$

   (d) $i(z) = \frac{z^2}{5} + \frac{5}{z^2}$

   (e) $j(t) = \cos(4t^2)$

   (f) $k(q) = 99q^{99} + e^{99q}$

2. Let $a \neq b$ be fixed real numbers. Prove the general formula

$$\int \frac{dx}{(x-a)(x-b)} = \frac{1}{a-b} \ln\left(\frac{x-a}{x-b}\right) + C.$$

3. Evaluate the following integrals. You will have to factor polynomials, use partial fractions, and divide polynomials by other polynomials.

   (a) $\displaystyle\int \frac{dx}{x^2 - 7x + 10}$

   (b) $\displaystyle\int \frac{9 - x^2}{x - 3}\, dx$

   (c) $\displaystyle\int \frac{dx}{x(x^2 + x)}$

   (d) $\displaystyle\int \frac{3x^2 - 2}{x - 4}\, dx$

   (e) $\displaystyle\int \frac{3x + 6}{x^2(x-1)(x-3)}\, dx$

   (f) **Bonus:** $\displaystyle\int \frac{5x - 1}{x^2 - 2x - 5}\, dx$

4. The *hyperbolic cosine* function $\cosh(x)$ is defined to be:

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x}).$$

   Find the arc length of the graph of $\cosh(x)$ on the interval $[-\ln(2), \ln(2)]$.